

# NAVAL POSTGRADUATE SCHOOL

**MONTEREY, CALIFORNIA** 

# **THESIS**

# APPLICATION OF TWO-SIDED MATCHING TO INTERNAL LABOR OF THE HELLENIC NAVY

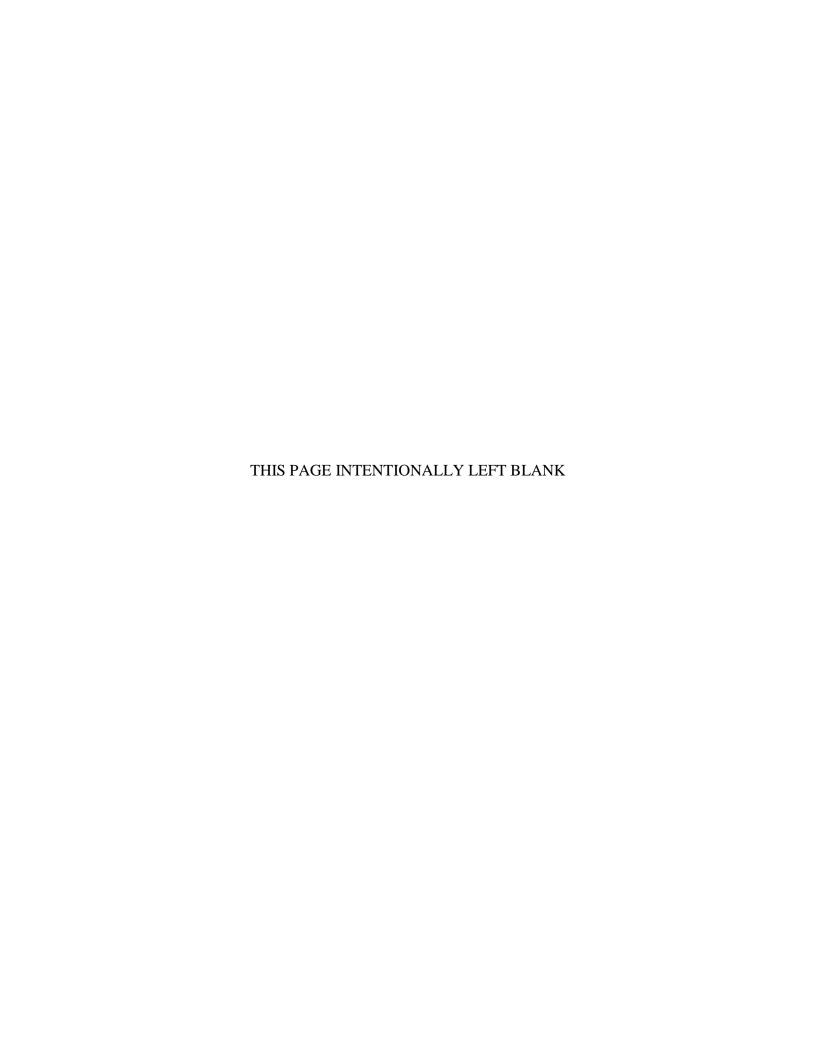
by

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March 2012

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1. AGENCY USE ONLY (Leave b	lank)	2. REPORT DATE March 2012	3. RE		ND DATES COVERED 's Thesis
4. TITLE AND SUBTITLE Applie of the Hellenic Navy		ded Matching to Interr	al Labor	5. FUNDING N	IUMBERS
6. AUTHOR(S) Paschalis Georgakoudis 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943–5000			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING /MONITORING N/A	G AGENCY NA	AME(S) AND ADDRE	ESS(ES)		ING/MONITORING EPORT NUMBER
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. IRB Protocol numberN/A					
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited			12b. DISTRIBU	<b>2b. DISTRIBUTION CODE</b> A	
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<b>14. SUBJECT TERMS:</b> Hellenic Navy's assignment process, human capital, performance, educational level, experience, two-sided matching theory, Priority mechanism, Deferred Acceptance, Top Trading Cycle mechanism, Qualitative- Quantitative evaluation			15. NUMBER OF PAGES 115		
					16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICAT PAGE		ABSTRAC	CATION OF	20. LIMITATION OF ABSTRACT UU

Standard Form 298 (Rev. 2–89) Prescribed by ANSI Std. 239–18

NSN 7540-01-280-5500

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# APPLICATION OF TWO-SIDED MATCHING TO INTERNAL LABOR OF THE HELLENIC NAVY

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Submitted in partial fulfillment of the requirements for the degree of

### MASTER OF SCIENCE IN MANAGEMENT

from the

# NAVAL POSTGRADUATE SCHOOL March 2012

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# **ABSTRACT**

The assignment process in an organization of any nature (i.e., military) plays a crucial role towards the direction of efficiency and effectiveness. A well-designed assignment process is a handy tool for the decision makers to fulfill the organization's goals.

This thesis explores the application of two-sided matching theory in the Hellenic Navy's assignment process. Three mechanisms (Priority-Deferred Acceptance and Top Trading Cycle) are chosen and developed, taking into account the magnitude of specific attributes like past performance, educational level, experience, officers' preferences and positions' requirements/priorities in order to achieve the most effective matching among officers and positions.

A fully detailed example and a few more cases are described, different scoring methods are used for evaluation, the results are compared and recommendations are provided so as to enable the matching theory to be suitably applied to the Hellenic Navy's assignment process.

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# **ACKNOWLEDGMENTS**

I would like to dedicate my thesis to my wonderful wife, Antigoni, and my beloved kids, Maro and Myrto; this journey started four years ago and it would not have a happy ending without your continuous encouragement, support and patience especially when the dream seemed to be lost.

Furthermore, Professor Noah Myung, thank you so much for your guidance and help in understanding material that I have not been taught, for being there for me whenever I needed your advice, and generally for your precious contribution in order to complete this thesis; I am really grateful to you.

Professor Benjamin Roberts, my appreciation to you started from the period that I had you as an instructor and it became greater during the preparation of this thesis. In such cases words are not enough; thank you very much.

Finally, I would like to thank my parents, Georgios and Mary, for their contributions in becoming what I am, and to express my admiration for the brave fight that they have fought side by side during the last thirty months against the "unbeatable enemy."

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### I. INTRODUCTION

#### A. BACKGROUND

Thus far, all officers of the Hellenic Navy have been able to offer their services in any position (key or not), given their level of experience and a few other characteristics. There is no significant evidence to justify potential inefficiency; however, the present placement procedure is likely simple and time consuming due to the lack of a structural mechanism. It puts more weights to only a few parts of each officer's available data (i.e., their preferences, the time that they served on board different types of ships and previous service in similar positions, etc.) and less to others that might be equally or more important. Additionally, positions with different requirements might be filled based on the same order of criteria even though the positions have different priorities. Moreover, subjective factors might be in place and closely related to the personality of those who make decisions about placements.

Through this thesis, three two-sided matching mechanisms will be described and applied to achieve the best matching between officers (with various level of education, experience, performance and preferences) and specific positions of various services of the Hellenic Navy. The above mentioned approach as a new systematic procedure could be a handy tool for decision makers to control for the gaps that already exist or even to replace the present placement procedure.

# B. PURPOSE

The purpose of this thesis is to achieve the most effective matching among officers (with specific attributes) and positions (leading or not) of the Hellenic Navy. The author will apply variations of a well-established matching/assignment model to the Hellenic Navy's preferences and the quality and preferences of naval officers. The goal of the study is to efficiently allocate officers to specific positions of the Hellenic Navy. Thus, taking into account the possession of a master's degree as an indicator of an advanced level of education, the kind of experience each officer has, their scores in evaluation reports and their preferences, the author tries to achieve the best match among

the preferences of those officers and the priorities/requirements that each position has. To do so, the methodology will be an exercise in evaluating various matching/assignment mechanisms.

## C. RESEARCH QUESTIONS

## 1. Primary Questions:

- How human capital factors like education, experience and performance level could affect the assignment procedure in the military environment?
- Which type of two-sided matching model should be used to best match quality and preferences of the Hellenic Navy's officers with position requirements/priorities in various cases?
- What are some practical implications of utilizing the two-sided matching models?

#### 2. Secondary Questions:

- How a specific position's preference list is created, taking into consideration its priorities/requirements and the characteristics of the eligible officers?
- Are the Hellenic Navy's evaluation reports a reliable tool for the assignment process? How could they become even more reliable?

#### D. SCOPE AND LIMITATIONS

The scope of this thesis will include the following:

- A literature review of specific human capital factors (possession of a master's degree as an indicator of a high level of education, experience and performance) that could affect the personnel assignment procedures in combination with personnel preferences and the priorities/requirements of specific positions.
- An attempt to turn each officer's performance and experience (that so far has been described in words) into numerical values so as to be used as crucial factors in the models.
- An attempt to review the existing form of the Hellenic Navy's evaluation reports so as to broaden the quality of information included and examine if there are any important factors that are missing.
- A presentation of the two-sided matching theory focusing on three specific mechanisms that could be applied in the Hellenic Navy's assignment procedure.

- The application of the three two-sided matching mechanisms in a full-detailed example in order to examine the results of using these mechanisms in the Hellenic Navy's assignment procedure. In addition, the presentations of more examples that cover various cases, check the strengths-weaknesses of the mechanisms, and compare the results so as to choose the appropriate mechanism for each specific case.
- The application of three main scoring methods (nine versions in total) based on mathematical formulas and statistical parameters in order to examine the effectiveness of each two-sided matching mechanism that are applied in the Hellenic Navy's assignment process examples.

#### E. METHODOLOGY

The methodology used in this thesis research will consist of the following steps:

- A literature review, based on research of textbooks and studies, will explain the magnitude of personnel-specific skills like level of education (in terms of the possession of a master's degree), level of performance (evaluation reports), and level of experience.
- A second literature review will present the two-sided matching theory, focusing and describing the function and further characteristics of three specific mechanisms.
- The two-sided matching theory will be applied to the Hellenic Navy's assignment procedure by taking into account the above mentioned human capital factors in combination with officers' preferences and positions' requirements; several examples will be developed towards that direction.
- The three main scoring methods will be applied using mathematical formulas and statistical parameters in order to examine the results of each matching mechanism and conclude the most appropriate one.
- The results will be analyzed, conclusions will be derived and recommendations will be made.

#### F. EXPECTED BENEFITS OF THE STUDY

This study will develop three two-sided matching models, based on game theory, so as to achieve the best matching of the Hellenic Navy's officer and positions by using specific criteria. The result will be the maximization of performance, and as an outcome overall efficiency of the organization. The model could then be used for key positions and for other positions too and become a handy tool for decision makers by replacing the existing time-consuming procedures.

#### G. ORGANIZATION OF THE THESIS

The thesis is organized as follows:

- Chapter II explains the magnitude of a high level of education (through the possession of master's degrees), experience (converted in numerical value) and performance (through evaluation reports) of an officer in order for these factors to be taken into consideration to the assignment procedure of assigning officers of the Hellenic Navy to key positions.
- Chapter III provides an overview of the two-sided matching theory (etymology- history- characteristics- advantages/disadvantages) and focuses on the application of three two-sided matching mechanisms: Priority, Deferred Acceptance and Top Trading Cycle.
- Chapter IV provides several examples for the application of the above mentioned mechanisms to the Hellenic Navy officers' assignment procedure, compares/analyzes the results of the three mechanisms, and highlights interesting points that are extracted.
- Chapter V proposes and applies three main scoring methods based on mathematical formulas and statistical parameters in order to evaluate the three two-sided matching mechanisms so as to contribute in the selection of the most preferable one.
- Chapter VI includes a summary, highlights and, conclusions, and provides recommendations for the application of two-sided matching mechanisms in the Hellenic Navy.

### II. THE ASSIGNMENT PROCESS

#### A. HUMAN CAPITAL - INTELLECTUAL CAPITAL

It is well acknowledged that human capital is the major factor of an organization. Human capital is not just the personnel, the people that work in/for the organization; it is also what these people offer and how they contribute to the organization's function. "Human capital is the collective value of the capabilities, knowledge, skills, life experience and motivation of an organizational workforce." <sup>1</sup> Since it is the factor that contains people it is the factor that can genuinely add value to the organization.<sup>2</sup>

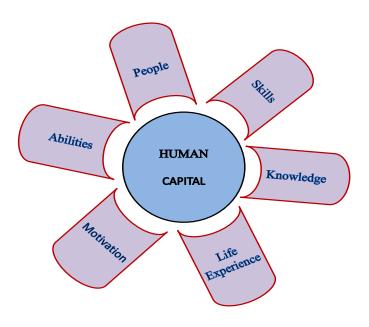


Figure 1. Human Capital

However, recent theories consider human capital as an element (the most important by far) of a more general term called intellectual capital. There are various definitions but all of them see intellectual capital as a conceptual platform from which to view, analyze, and hopefully quantify the intangible assets of an organization. The other primary elements of intellectual capital are social and organizational capital. According

to Baron and Armstrong, "the tripartite concept of intellectual capital indicates that, while it is individuals who generate, retain and use knowledge (human capital), this knowledge is enhanced by the interactions between them (social capital) to generate the institutionalized knowledge possessed by an organization (organizational capital)."<sup>2</sup> Any established manpower system must be examined and thought of in connection to these elements.

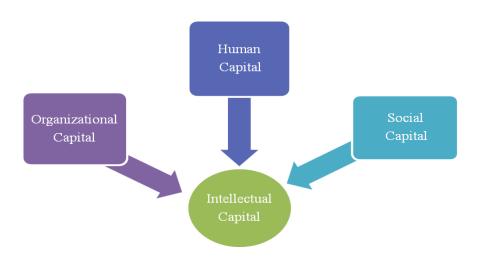


Figure 2. Intellectual Capital

Throughout the thesis, the author will focus on human capital because it is the factor that includes personnel and other characteristics that are the core of the assignment process.

The management and use of an organization's human capital is a crucial factor for its efficiency and its effectiveness. Efficiency and effectiveness are often considered synonyms, but they have different meanings when applied to the process of management in an organization. According to Mathis and Jackson, "efficiency is the degree to which operations are done in an economical manner" while "effectiveness of an organization is a measure of the ability of a program, project, or task to produce a specific desired effect or result that can be measured." In simple words, efficiency is doing things right and effectiveness is doing the right things.

#### B. ASSIGNMENT OF PERSONNEL

Assignment is the process according to which the organization's personnel is employed in such a way that covers both the organization's needs and the personnel's preferences. A well-designed and easy to use assignment process is a handy tool for the organization's decision makers; the results are beneficial and towards the direction that fulfills the organization's goals. If an employee is satisfied with his/her assignment, this can cause an increase to his/her mood, morale, performance, productivity, and even retention. Everything seems to improve and the goals tend to be met. In reverse, a potential placement of employees in positions where the requirements do not meet their skills may have undesirable results and suboptimal outcomes.

Therefore, efficiency and effectiveness are the major objects of an assignment process. An assignment process is said to be efficient when it matches appropriate, well-trained, and skillful personnel to specific jobs. On the other hand, effectiveness in the assignment process has to do with timeliness. An assignment process is said to be effective when the personnel occupy specific jobs at the right time.<sup>3</sup> Both parameters can be seen and evaluated from the employee's and position's or organization's point of view.

The design and the application of an appropriate assignment process inside an organization is not a simple procedure and it depends on various parameters. The number of employees, their skills/abilities/knowledge/educational level/experience/preferences (human capital in general) in combination with each position's specific requirements, organizational culture/strategy/partial policies, and multiple other criteria that have to be met, even in a minimum degree, prove the complexity of an assignment process and indicate the necessity of its existence. The above mentioned complexity may lead to non-optimal assignments, and it is possible for human error to affect the procedure and thus the consequences will be negative for both for the organization and personnel. The assignment process in a military organization differs substantially from that of a normal market organization. A plethora of unique qualitative and quantitative features makes the process more complex and more difficult to be "solved" in an optimal manner; some of these features are the following:

- Hierarchy plays a catalytic role in every aspect of military life.
- The participation of high rank/senior/junior/petty officers and personnel in general in the assignment process is compulsory.
- Large groups of personnel members have to be assigned simultaneously.
- The process takes place several times during the year, each time for different categories of personnel.
- Manpower data is huge and information load is increased day by day.
- Preferences (from positions and personnel) and human needs have to be taken into consideration.

Moreover, it must be taken into account that in most cases the military assignment process, including manpower data collection/managing and the matching procedure, takes place manually and without using a decision support system (DSS). Thus, the "problematic" issue concerning the military assignment process is revealed clearly. That does not mean that the result of a current assignment procedure is not satisfactory, but most of the times it is far to be considered as the optimal one.

#### C. THE HELLENIC NAVY'S ASSIGNMENT PROCESS

The purpose of the thesis is to apply two-sided matching theory in the Hellenic Navy assignment process in order to achieve the most effective matching among the officers and key positions. Therefore, it would be useful to present the current assignment process briefly but important to mention from the beginning that the process is not known to personnel with full details.

The process starts one year before the implementation of assignments when each officer submits by mail to the Department of Personnel his/her annual preference list which contains the positions where he/she would prefer to be assigned. Also, this list gives each officer the potential to include further information, whether very important (like report of personal issues, acquisition of a master's degree or a foreign language certificate) or less important (like change of home address, etc.). Officers' preference lists are just a part of the manpower data that the Department of Personnel keeps in its possession. Some of the data that needs to be taken into account by the detailers during the assignment process are annual or special performance evaluation reports, certificates

of educational level, records of previous positions, and punishments for disciplinary reasons. Then, the manpower data are processed and finally the matching among the officers and positions takes place.

Parameters that act as a barrier towards the assignment process's direction for an optimal result are the following:

- The enormous volume of manpower data.
- The lack of a manpower database that would give the detailers the opportunity for direct/fast access and managing of significant information.
- The fact that the assignment process takes place not just once but a few times every year; each time the process concerns officers with different characteristics.

The results of facing these parameters is an extensive time-consuming procedure that lasts many weeks and even months, significant effort from the detailers, and data that has not been processed in the optimal manner or been taken into consideration at all. Thus, it is logical that the final result is often a suboptimal one for both officers and positions.

### D. HUMAN DECISION MAKING IN THE ASSIGNMENT PROCESS

The above mentioned barriers during the assignment process are caused by factors that have to do with the structure, organization and followed procedures through personnel issues processed in the Hellenic Navy. However, there is another factor, equivalent or even more important, that affects the result of the assignment process, —the human factor and specifically in this case human decision making.

Decision making is a conscious process of making choices among alternatives with the intention of moving toward some desired state of affairs.<sup>4</sup> Having the heavy weight of making important decisions (like assignments) the decision-makers (detailers) try to decide based on pure logic or rationality. According to Mc Shane and Von Glinow,<sup>5</sup> the rational choice decision-making process contains the following steps:

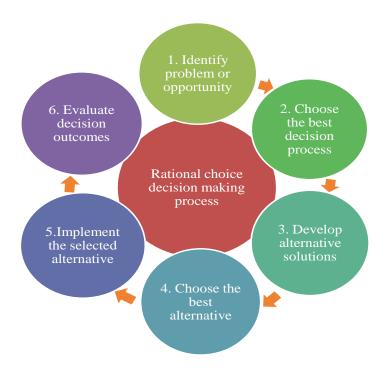


Figure 3. The Rational Choice Decision-making Process<sup>5</sup>

The "rational choice decision-making cycle" seems to be logical but its application faces various problems.<sup>5</sup>

- The "cycle" assumes that the decision-makers are efficient and logical information processing machines.
- It focuses on logical thinking but it does not pay attention to emotions, a factor that affects the decision-making process significantly and in many ways.
- Some steps are based on appropriate and accurate information, thus
  incomplete information processing leads to results that are below the
  expected ones.
- Decision-makers generally do not try to maximize the outcome; that is they make an acceptable decision, without looking for the solution with the highest pay-off. That happens because they cannot develop and evaluate all the alternatives, so they select an option with a "score" above a subjective minimum limit considering that it could satisfy the needs.

• Stakeholders (superior and subordinate officers) with vested interests in the assignment process try to "frame" the situation by influencing the decision makers through various ways.

Applying decision-making theory to the Hellenic Navy assignment procedure and taking into account the above mentioned barriers in combination with the size of the organization, its complexity, the absence of a manpower database and decision support, the author realizes that an optimal result is difficult to be achieved. Thus, the Department of Personnel is likely to be satisfied with just a "good" one. However, there are ways to improve the process from its first step until the final step. The utilization of advanced technology (networks, internet, etc.) in order to simplify and eliminate the time needed for the collection and managing of manpower data would be a useful tool towards improving the process. Furthermore, the manning of the Department of Personnel with more staff in order to limit the workload and pressure that derives from the importance of its tasks could be proven a practical measure.

However, the most important direction in which the assignment process must be headed is the elimination of issues that affect it in a negative way, like ad hoc, favoritism etc. A way to eliminate those issues is the adopting and application of a specific matching mechanism, a mechanism that would take into consideration the most crucial characteristics/skills of each officer, his/her preferences, and the requirements of the positions that must be covered. The detailer, by following predefined steps and specific rules through a mechanism, does not have the opportunity to "allow" such actions/behaviors (coming from emotions or any other factor) that influence the assignment process. Perhaps such issues will not be completely eliminated this way, but at least they will be reduced.

For that reason, two-sided matching theory was chosen and three specific mechanisms will be developed and applied in The Hellenic Navy's assignment process. In order to simulate a real situation as much as possible, the author assumes that fifteen officers are eligible to cover ten positions. The detailer has an extensive amount of information about the officers and thus it will make it difficult to manage "comme il faut" (that means, in an appropriate way); therefore the author proposes that he/she should take

into account just the most important ones. The question that emerges is which of all the characteristics play a significant role in the assignment process?

There are major and secondary characteristics. A major focus would consider the officer's performance that is imprinted in the performance evaluation reports, the educational level in terms of possession of a master's degree, and experience. Some of the secondary characteristics could be important in some cases like knowledge of appropriate foreign languages in case of a position abroad. In general, their magnitude cannot affect the final assignment because positions' preference lists will be created and the officers that fall short will be positioned last in the order for the specific positions (i.e., an officer that does not speak French will be positioned last in order for the Paris Naval Attaché's position).

#### E. MAJOR CHARACTERISTICS

#### 1. Performance

Various definitions exist for performance, concerning different types of activities (like job, task, academic, and financial performances, etc.). The type of performance on which the author focuses on in the thesis is job performance. Job performance refers to the way that an employee performs his/her work. There is no doubt that an employee's performance is the characteristic with the greatest importance because it is strongly related to the organization's outcome and success. Better performance increases the possibility for success.

There are several characteristics that an employee must possess in order to perform his/her work effectively and successfully (i.e., ability, motivation, intelligence, etc.). These characteristics are called selection criteria. Selection criteria are set according to the hierarchy level of an employee in an organization; i.e., for a managerial job performance, more specific selection criteria like "leading and deciding," "supporting and cooperating," etc. are needed.<sup>1</sup>

However, even though job performance could be considered as individual's behavior (a group of employee characteristics and actions) it is not to do just with the employee. The role of the supervisors in an employee's successful job performance is

crucial too. Supervisors must illustrate the job description to each employee, explain thoroughly the position's requirements and standards that the employee has to meet, and set the strategy, goals and objectives clearly even on a daily basis if they think that it would be helpful. This procedure or specific parts of it must be repeated if the supervisors estimate that it is necessary. This is the first part of their role. The second part, which is the most difficult to be implemented and the most important, is performance measurement. Performance measurement is the collection of information regarding the performance of an individual (employee, manager, etc.) or group of individuals. Measurement of an employee's performance allows the supervisor to:

- Evaluate how well the assigned job is being done.
- Check/control if the organization's strategy is being followed.
- Interfere in cases of declination of the organization's goals.
- Motivate by giving new incentives.
- Reward by promoting or offering material goods.
- Improve what he/she realizes needs to be improved.

However, what is the best way of measuring performance? What elements must be taken into account in order to achieve a fair and realistic representation of an employee's performance? The only way of measuring performance is to use statistics that concern the collection of data, analysis, and interpretation of results. It is obvious that performance cannot be measured accurately; this is because it contains intangible characteristics and characteristics that cannot be measured directly but just based on indirect observations, estimations or assumptions. Most of the organizations use an annual performance report where the supervisors grade various characteristics of an employee's tangibles and intangibles, describe his/her achievements, compare them with the organization's expectations, write comments, and make conclusions and recommendations. This method allows the leaders to have a global view of each employee's abilities/personality/overall performance, and a helpful database for future use.

Figure 4 illustrates a mechanism of good job performance that contains predictors of selection criteria and elements of good job performance. The author needs to mention

that predictors of selection criteria are measurable or visible indicators that in case of possession an employee is likely to perform his/her work better. It is a factor strongly connected with decision making and very important for the development of his examples in the following chapters.<sup>1</sup>

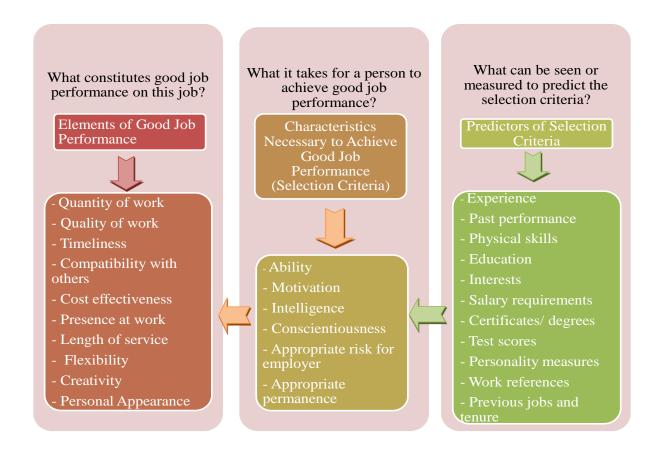


Figure 4. Job Performance, Selection Criteria and Predictors <sup>1</sup>

As far as the Hellenic Navy is concerned, a method similar to the abovementioned one has been adopted in order to measure the performance of its staff. Once a year under normal circumstances (or more than one under exceptional circumstances) the direct supervisor (first evaluator) fills out a performance evaluation report for each of his/her subordinate officers. Then, the report is shown to the person being evaluated, who signs it as a proof of acknowledgment or declares his/her objections in writing. Finally, it is forwarded to the second evaluator for the final signature and submission to the Department of Personnel.

The form is a standard one with a specific pattern. It is an impressively well-designed form with various fields that must be filled in by the evaluators covering all the aspects of an officer's personality and performance. Physical, mental, psychic, administrative, professional, special and ethical skills are the general fields on the form, while each of them is divided into many subfields. Furthermore, there is space for general/special comments and a final assessment (a number between 0–100) that indicates the general ability and performance of the person being evaluated. If this fully detailed evaluation form is filled out in an accurate manner, it provides a complete image of the person being evaluated performance for the specific time period.

However, there is a deficiency that affects the credibility of the procedure in a negative way; the procedure does not take into consideration neither the personality nor the perceptual errors of the evaluator. There is no referral at all to the evaluator. That means that the comparison between two officers is based just on absolute numbers no matter who the evaluator was in each case. For example, assume that two officers of the same rank, O<sub>1</sub> and O<sub>2</sub>, serve in different positions (i.e., different ships) and must be evaluated by their supervisors (commanding officers),  $S_1$  and  $S_2$  respectively.  $S_1$  is a very strict and demanding supervisor who grades his subordinates so far with an average of 90/100 (best 93/100 and least 85/100); S<sub>2</sub> is a supervisor that follows a different way of leading and he grades his subordinates so far with an average of 98/100 (best 100/100 and least 96/100). It is assumed that S<sub>1</sub> grades O<sub>1</sub>'s general performance with 94/100 and S<sub>2</sub> grades O<sub>2</sub>'s general performance with 96/100; in absolute numbers, O<sub>2</sub> had a better performance than O<sub>1</sub>. However, if the issue is investigated thoroughly, it is realized that  $\mathrm{O}_1$  received the best grade that  $\mathrm{S}_1$  ever marked and  $\mathrm{O}_2$  received the worst grade that  $\mathrm{S}_2$ ever marked. That means that O<sub>1</sub> did not perform better than O<sub>2</sub> and his 96/100 compared to S<sub>2</sub>'s 94/100 does not imply a better performance.

This problem is not caused just from the evaluator's personality, principles and way of leading. It is also a matter of perception and the perceptual process. "Perception is the process of receiving information about and making sense of the world around us." After that, the receiving information is filtered through an imperfect perceptual process where selecting, organizing and interpreting of information takes place. There are many different perceptual processes, while each of them creates bias (called perceptual error). Suggestively some of those perceptual processes are the following: <sup>5</sup>

- Selective attention: focusing on some information while ignoring other information.
- Mental models: visual images in the mind that represent the external world.
- Self-serving bias: attribute preferable outcomes to internal factors and failures to external ones.
- Halo effect: the impression of a person based on a specific characteristic affects the impression of other characteristics.
- Primary effect: an impression of someone is formed based on the information first received about him/her.

Perceptual processes cannot be avoided but there are ways to eliminate their negative results (biases). Therefore, as far as evaluation reports are concerned the author looks for a different way to proceed, taking into account the evaluator's characteristics (including potential biases) in order to reflect the performance as pragmatic as possible.

A good solution to the above mentioned problem could be the performance evaluation system of the U.S. Marines. According to that system, all grades (in a 0.0 to 5.0 scale) of the specific evaluator in the past are used in order to create a percentage context where the greatest grade would be 100% and the lowest one would be the respective percentage (i.e., if the greatest grade is 4.8 and the lowest 3.6, the 4.8 reflects the 100% of the context and the 3.6 reflects the 75% of the context). The grade that corresponds to the performance of the person being evaluated (i.e., 4.2) is put in the context (87.5%) and represents an actual performance comparable rather fairly to the other ones.

Finally, a minor issue is the accurate (as possible) estimate of the number that reflects the general performance of the person being evaluated. There is no mathematical or statistical formula that is used in order to calculate it, but there seems to be a number that represents the general impression that the evaluator has for those being evaluated. However, the number that reflects the general performance should be derived from the grades in the partial fields of the evaluation report; if not from all at least from the most important of them (the choice will take place according to Department of Personnel criteria). Then, a mathematical or statistical formula (i.e., arithmetic mean or standard deviation) could be used for the final calculation.

## 2. Educational Level (Possession of a Master's Degree)

Education is likely to be the most important mechanism that contributes to the acquisition and development of human capital. It is the mean for an individual to acquire new skills and knowledge that will lead to important private and social benefits/returns. That means education is considered as an investment that is "spending now and expecting to gain in the future."

The amount of education that is acquired by an individual/worker/employee has a significant impact to his/her personality, work, and the labor force in general. A more educated individual/employee is more able to:

- Absorb new information.<sup>6</sup>
- Learn-by-doing.
- Be trained.
- Adopt new technologies.
- Develop innovation.
- Learn and execute successfully complex tasks.
- "Develop features like work habits, awareness of time, dependability".

As a result, a more educated individual/employee increases his/her personal productivity. This implies:

• The demand and achievement of a higher wage in his/her professional life.

• The increase of labor participation, the improvement of labor force as a factor of production, the decrease of unemployment probability, and the decrease of turnovers as far as the labor force in general is concerned.8

Also, there are effects (private and public) that have nothing to do with the market. Researchers have proven that a more educated individual tends to protect his/her investment (education) by taking care of his/her health issues or by taking preventative measures more often than a non-educated individual in order to avoid unpleasant events in health matters. Furthermore, education enables people to be better parents, children, neighbors, citizens and voters contributing to the institution of a stable and democratic society. Finally, the criminal activity in an area is conversely proportional with the educational level of its residents. That is, the average crime rate is decreased in areas with a high percentage of educated residents.

However, a question that derives from this topic is if all these implications lead to increased economic growth. The answer in that question is neither positive nor negative. A shortage of educated people may lead to decreased economic growth, but it is not certain that the influence of more educated people will guarantee the expected result. The reason is that the relationship between education (and human capital in general) and economic growth is highly conditioned by the quality and distribution of education in the labor force and the economic structure of the country. Investing in more and better-distributed education in the labor force helps create conditions that could lead to higher economic growth, but this is by no means sufficient. It is also necessary to adopt policies that lead to the creation of diversified, dynamic, and competitive sectors capable of absorbing the more educated labor force to translate education (human capital in general) into higher economic growth.<sup>7</sup>

Education, having the shape of an investment, implies not only benefits but costs too that one hopes to compensate over a period of time. Costs of education are divided in two parts—tangible or direct costs and intangible costs. The first part includes costs that can be measured and quantified like tuition, foregone earnings/wages, studying material

and other expenses concerning schooling; the second part includes costs that cannot be measured like time spent, psychic losses (that occur due to the difficulties that one might be facing), etc.<sup>8</sup>

Therefore, the big question is whether education is a good investment or not. This is a question that concerns both individuals and government/company decision-makers; individuals wonder if education will increase their monetary and non-monetary benefitswhile policy/decision makers have to evaluate if the social/company benefits of the provided education will outweigh the costs. There are a few methods (i.e., net present value method, internal rate of return method) developed to answer this question, but the problem is that they analyze only the monetary part by using statistics and data. Furthermore, the delay in receiving the returns in comparison to the costs, the above mentioned prerequisites and other important factors like innate ability, psychic costs, etc. that are biased and difficult to be measured make the final result not completely accurate or even that useful.

The following figures illustrate two examples concerning the monetary effects of education. In the first example (Figure 5), two individuals at the age of eighteen follow different directions; individual A begins to work while individual B goes to a university. Thus, two earning streams appear; earning stream A of the high school graduate which starts to rise immediately but not so high and earning stream B of the university graduate which starts with a negative income for the first four years but after that it takes off and rises above stream A.<sup>10</sup> It is obvious that in the future the earnings of individual B will bypass those of individual A (the difference of earnings is called gross benefit) and that happens due to the educational benefits received by individual B. Individual B can achieve higher wages than individual A.

In the second example (Figure 6), both individuals A and B are university graduates and begin to work at the age of twenty-two. It is assumed that for various reasons employee A has a slightly higher wage than employee B. At the age of thirty, employee B is chosen by his company's decision makers to acquire a master's degree

concerning his subject. Two years after, for the same reasons as in the previous example, the wage of employee B will rise and will be higher than that of employee A.

The author has to clarify that the above mentioned examples (and their graphs) are presented in order to show the potential monetary benefits of an employee (or a student) that receives additional education in comparison with another employee (or a student) that he doesn't. Of course the whole task is not so simple, an investment like that needs to be analyzed in depth using the Net Present Value or other evaluation method and taking into account parameters like discount rate, cash flow or even opportunity cost in order to have accurate and secure conclusions for the investment.

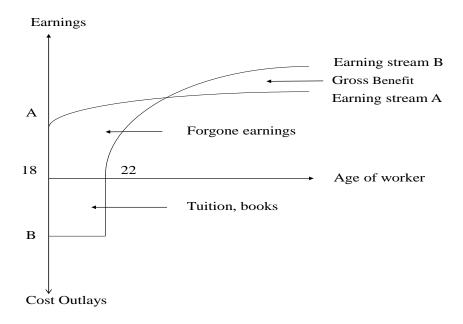


Figure 5. First Example in Monetary Effects of Education

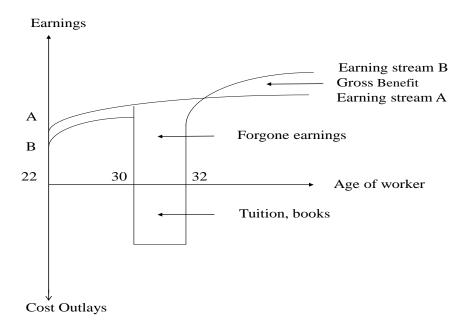


Figure 6. Second Example in Monetary Effects of Education

As far as the Hellenic Navy is concerned, the provided education to officers takes place in various time periods of their career. The first step takes place during their training in the Hellenic Naval Academy and it lasts four years. It includes teaching and a deep dive into a wide area of technical, theoretical and specialized courses/labs by taking advantage of technology and the presence of referable professors. After graduation from the Academy, an officer is considered to have the same educational level as a graduate from a top-ranking educational institute. After that, the provided education has the form of mandatory short-term courses (they usually last few months) that are professionally orientated. That means that they include updates to an officer's knowledge and focus on his/her work in various positions (ships, repair stations, etc.) as well as a general view in war history, geopolitical strategy and global issues.

Moreover, the Hellenic Navy offers to its officers the opportunity of acquiring a master's degree in various sections (like information technology, management, computer science, shipbuilding, mechanical engineering, etc.) in the U.S., Great Britain, or at a domestic university. The acquisition of a master's degree gives both the officers and the Hellenic Navy all the advantages that were discussed above. The percentage of the

officers that achieve to acquire a master's degree having the support of the Hellenic Navy in various ways (like paying the tuition, allowing a time period of two years off, etc.) is ranges between approximately 20–25%.

Comparing the acquired education of the market's labor force with that of the Hellenic Navy's internal labor force one can refer to differences and similarities. The implications that concern the individual's professional life (demand of higher wage) and the labor force in general (increase of labor participation, improvement of labor force as a factor of production, decrease of unemployment's probability, economic growth) do not apply in the Hellenic Navy due to the special military environment. In reverse, the implications for the further effects of education (parental behavior, upbringing, crime rate, health issues, etc.) apply the same way to the Hellenic Navy's officers that have achieved to acquire a higher educational level. The obvious conclusion is that the Hellenic Navy's decision makers believe that the acquisition of a master's degree by an officer has beneficial effects, both for the officer and the Hellenic Navy even though the expenses are high. They believe that the benefits, through the increase of productivity, will compensate the Hellenic Navy for the losses that come from direct costs and foregone earnings, and for that reason the Hellenic Navy continues to contribute towards that direction.

However, how can the author take into account the possession of a master's degree if it is to be used as a criterion for the assignment process? Whether an officer possesses a master's degree or not can be represented as a binary code where 1 represents an officer that possesses a master's degree and 0 represents an officer without a master's degree.

So far, the author analyzed the past performance and education as characteristics that affect the assignment process; the last (but not least) major characteristic that the author is going to evaluate is experience.

#### 3. Experience

Experience, in general terms, is a concept that includes accumulated knowledge and skills gained by doing a job, an activity or being in a lot of different situations.<sup>11</sup>

Experience is an "advantage" that plays an important role in the hiring process. Some of the reasons that contribute to the importance of its role are the following:

#### An experienced candidate

- Already has valuable insight by working in different environments under different circumstances, having various duties, and trying to achieve goals of a different nature.
- Has already developed to a satisfactory degree various skills (i.e., communication skills) that an employer expects from his/her employees.
- Has a self-confident personality and it is easier for him/her to make a difficult decision.
- Has already worked in a team and cooperated with various types of people, supervisors and subordinates.

These are some reasons for which organizations hire on the basis of work experience; in general terms that happens because employers expect experienced workers to perform better than non-experienced ones. However, do their expectations come true? Are a firm's expectations about the benefits of hiring an experienced employee always met? In order to answer that question, the author has to divide experience in two subcategories—prior work experience and prior related work experience. Prior work experience provides the opportunity for knowledge acquisition, and prior related work experience provides not only opportunity but also greater potential applicability of that knowledge to the new context. Prior work experience is likely to have a neutral effect on performance while prior related work experience has both a positive and a negative effect; a strong positive one because prior related work experience may increase performance (indirectly) via related knowledge and skill; in few words, a firm that hires an experienced (in related tasks) employee also brings additional precious human capital. However, there is also a direct negative effect due to behavioral and cognitive rigidities.

That does not mean that all work experience generates useful knowledge and skills when applied in a different environment, similar to the old one or not. This is easier to achieve in a case of jobs of a similar nature where the acquired relevant knowledge and skill can be applicable to performance in a new environment; however, in a case of jobs with unrelated work activities this is likely to be more difficult.

Experience in military service can be also separated into prior work experience and prior related work experience. The difference from the market case is that prior work experience in the military environment has a positive effect in performance even in a case where an officer is assigned in a position of a totally different nature (i.e., from a frigate to an office of the General Staff). Of course, the effect is not the same as the effect of prior related work experience, but it exists and it is helpful for the officer. As far as the Hellenic Navy is concerned, the majority of officers have a similar level of "provided" experience. It is the Hellenic Navy's policy that all officers (except those of special services like submariners, etc.) are assigned in different positions almost every two years; therefore, the officers have the opportunity to acquire miscellaneous knowledge and develop their skills by serving in different positions. However that is not enough; the acquisition of experience is not just a matter of opportunities but it also depends on the way that each person tries to grab the provided opportunities. It is also based on the desire and effort that an officer makes in order to acquire the experience. Thus, even though the officers have almost the same opportunities, their level of experience differs.

The measurement of an officer's experience, and especially the conversion of his experience into a number, is not easily done in a fair and objective way because experience is an intangible feature and has to do with the positions in which the officer has served in the past. In addition, it is not necessary to measure experience under normal circumstances. However, such a procedure would be helpful to take place during the assignment process and the extracted "number" that represents an officer's experience would be useful as a factor that could be taken into account. As an example, assume an officer who has served six years in a frigate and two years in a destroyer; this officer has two different "countable" levels of experience, one level that represents his experience in frigates and another one that represents his experience in destroyers. Thus, during the assignment procedure the Department of Personnel must numerate a candidate officer's experience according to the position that must be covered.

A potential method to measure an officer's experience takes into consideration his/her past service both in a similar (or even the same position but under different duties) position and in a position of a different nature but having the same duties. For example,

in order to measure the experience of an officer who is a candidate for the position of Vice Commander in Frigates Command his/her past service in the specific command or other command of a similar nature (like Destroyers Command) will be looked at as well as his/her past service as a Vice Commander anywhere else. Weights must be taken into account to distribute in a fair way the services that are included in the measurement of experience. For example, assume that a candidate for the Frigates Command Vice Commander position served in the past three years in the specific command as a staff officer, two years in the Destroyers Command as a staff officer, and one-and-a-half years as a Vice Commander in another position. These services cannot have the same weight; the candidate's previous service in the same staff seems to be more important from the other two. Thus, a potential mathematical formula for the calculation of experience could be: E = 5x3 + 3x2 + 2x1.5, where 5/3 and 2 are potential weights for each type of service. The Department of Personnel must specify the weights that will be used in order to calculate the officers' experience during the assignment process.

So far, the author has analyzed the three major characteristics that are predictors of selection criteria and must be taken into account in the assignment process—the performance, educational level and experience. The Department of Personnel will draw the table that contains the candidate officers' scores in each characteristic and will set the priority of requirements for each position that has to be covered. Therefore, the officers will be classified according to their score in the first requirement (from higher to lower), and in a case of a tie the officer with the higher second characteristic will have the advantage. In the extreme case of a second tie, the officer with the higher third characteristic will be chosen. Using this methodology, the Department of Personnel will create the positions' preference list and in combination with the officers' preference list (that is submitted by them annually) and the application of two-sided matching theory, the assignment process will take place.

For example, assume that the Department of Personnel set the priority of requirements for a specific position as follows: performance and possession of a master's degree. The candidate officers and their characteristics (extracted as described above) are shown in Table 1.

Table 1. Example for the Creation of a Position's Preference List

Officers	Performance	Possession of master's degree	Experience
O1	96	1	15
O2	98	О	18
О3	96	О	22
O4	97	1	17
O5	98	0	23
O6	98	1	20

It can be seen that officers O2, O5 and O6 have the same performance score (98), so the tie will be broken by the second characteristic (possession of a master's degree). Officer O6 is the only officer that possesses a master's degree, thus he is the first choice in the preference list. Officers O5 and O2 do not possess a master's degree so the tie will be broken by the experience score; Officer O5's score is 98 while officer O2's score is 95, thus, officer O5 is the second choice and officer O2 the third. Similarly, the fourth choice is officer O4 and the tie between officers O1-O3 is broken by the second characteristic (officer O1 possesses a master's degree while officer O3 does not). Consequently, the final position's preference list would be as follows: O6, O5, O2, O4, O1 and O3.

#### III. TWO-SIDED MATCHING THEORY

#### A. BACKGROUND

Etymologically, the term "two-sided matching market" refers to the presence of two distinct groups of agents (two-sided) and the bilateral nature of exchange where every agent of each group seeks to be matched with his/her most preferred agent from the other group (matching).<sup>13</sup> The process for generating the matching between the agents of the two groups is called a two-sided matching mechanism.

The need for the application of two-sided matching started around the turn of the last century and it concerned the internal medical market and the assignment of medical students to hospital residency programs. Many difficulties appeared as the years passed<sup>14</sup>, including the following.

- The hard competition between hospitals because the number of positions offered was greater than the number of available students.
- Students were unhappy because they were pressed to accept offers before their alternate status was resolved and without having the potential to wait until the last minute for a more preferable offer.
- Hospitals were also unhappy because they faced many last-minute rejections from their preferable candidates and the alternate ones had in the meantime already accepted other positions.

The National Intern Matching Program (NIMP) was the first two-sided matching mechanism that applied in practice in the 1951–52 U.S medical market, even though it faced the above mentioned issues. Each student that was in the final year of medical school was interviewed for some programs and each program interviewed some students. Afterwards, the students ranked the programs according to their preferences and vice versa. Finally, each student was assigned to a specific program while each program covered its available positions with a specific number of students. In a case of unfilled positions, those could be covered by foreign students, and in a case of unmatched students they could seek matching individually. Over the years the program was developed trying to solve the apparent problematic issues, and nowadays it is still in use with a slightly different name (National Resident Matching Program- NRMP). <sup>13</sup>

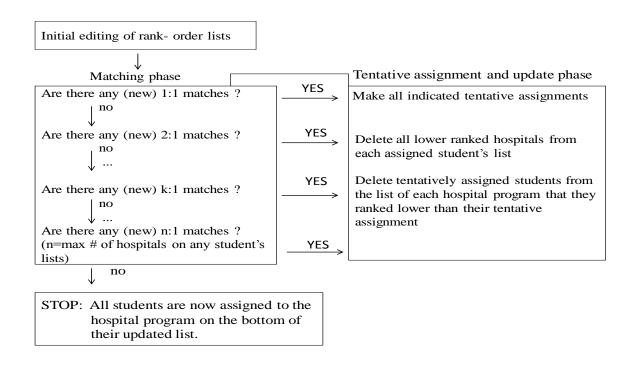


Figure 7. The NIMP Algorithm<sup>13</sup>

The first memorable theoretical study of two-sided matching belongs to Milton Friedman and his 1955 article, "The Role of Government in Education," which refers to the "school choice" topic. In this article, the term "school voucher" appears for the first time and it is an early step towards the direction to give parents the opportunity to choose the school their child will attend. In the last fifteen years, the usual practice of school choice according to geographical locations has changed. Miscellaneous mechanisms (with their advantages and disadvantages) were developed, leading to the introduction of various choice programs that allow parents to pick schools for their children within or outside their school district.

Furthermore, over the years the school choice topic and its literature is spread to other closely related topics like the college admission topic (namely, the assignment of candidates to colleges) and the allocation of dormitory rooms topic (that is, the allocation of on campus housing facilities to students). The first case as it was named and described by Gale and Shapley became very popular and was studied extensively, analyzed thoroughly and applied successfully in the college labor market.<sup>16</sup> It is close to the school

choice topic but even closer to the medical market problem. That happens because in college admissions and the medical market, students and schools/hospitals are both agents with preferences and priorities, while school choice schools do not have preferences but wait to be chosen by the students <sup>17</sup>.

In the second case, Hylland and Zeckhauser propose a mechanism, known as random serial dictatorship, where the students are ordered by chance (single lottery) and the first student is assigned to his/her top choice, the second student to his/her top choice among the remaining slots, and so on. The specific mechanism is effective and can accommodate any hierarchy of seniorities, but it cannot be applied in school choice or college admission problems because the priority ordering of a student is different from school to school.<sup>17</sup>

As is obvious, the education market was the dominant field for the development of two-sided matching. Nowadays, various two-sided matching mechanisms/models are applied in a wide variety of real life markets (besides the education one) like the labor market, including various types of workers and firms, public employees and public positions, etc.). It is also applied in social processes (like marriageable men and women), providing significant solutions to the assignment problem.

#### B. CHARACTERISTICS

There are two basic matching models, differing only in whether the agents of each side prefer to be matched with (at most) one agent of the other group (one-to-one model) or with many agents of the other side (many-to-one model).

The one-to-one model is known as the marriage model. The males represent the first group and the females the opposite group. Each person has a preference list that contains members of the opposite group with whom he/she would like to be engaged. Each agent is matched with one agent of the opposite group or remains single rather than be engaged with an undesirable agent. A significant remark is that the group that does the proposing must be assigned, because the matching could be different when men propose than when women propose.<sup>18</sup>

Example No 1 Men	Women	Potential Matching μ <sub>1</sub>
$Pm_1(w_2, w_1, w_3)$	$Pw_1(m_2, m_1, m_3)$	$m_1 \bigvee w_1$
$Pm_2(w_1, w_3, w_2)$	$Pw_2(m_1, m_2, m_3)$	$m_2$ $w$
$Pm_3(w_3, w_1, w_2)$	$Pw_3 (m_3, m_1, m_2)$	$m_3 \longleftrightarrow w_3$

Figure 8. The Marriage Model

The many-to-one models are most frequently applied to the labor and education markets and are regarded as entry-level markets too; that is, in many cases the agents from the one side are entering the market for the first time. An example is the college admission model where each individual agent (college) of one group seeks to be matched with many agents (students) of the other group. In this type of model, each agent of the group of firms/colleges/schools seeks to be matched with a number of agents with similar characteristics and skills from the opposite side. Therefore, its preference list must be defined through a strategy and not just over individual workers/students but over a group of students that covers its requirements.

Example No 2	<u>Schools</u>	<u>Students</u>	Potential Matching μ <sub>2</sub>
	$Ps_1(i_2, i_1, i_3)$	$Pi_1(s_2, s_1)$	$s_1 \longrightarrow i_1$
	$Ps_2(i_1, i_3, i_2)$	$Pi_{2}\left( s_{1},s_{2}\right)$	$S_2$ $i_2$
		$Pi_3(s_2, s_1)$	i <sub>3</sub>

Figure 9. The Many-to-One Model

There are three important things to really consider in the two-sided matching theory. First we will consider stability issue, one of the most important conditions in order to ensure the success of a matching assignment is stability. "Stability is used instead of the Nash equilibrium and as the main solution concept." This does not mean that markets with unstable matches cannot operate, but the possibility is definitely eliminated. So, according to Roth and Sotomayor, <sup>13</sup> and as far as one-to-one matching is concerned, "a matching is stable if it is not blocked by any individual or any pair of

agents;" an unstable matching takes place when a pair of agents that prefer each other as a partner is not matched and the specific pair is called a "blocking pair."

For example, there are two men  $(m_1, m_2)$  and two women  $(w_1, w_2)$  and we assume that the matching assignment is  $m_1$ - $w_2$ ,  $m_2$ - $w_1$ . If  $m_1$  prefers to be matched with  $w_2$  (rather than  $w_1$ ) and  $w_2$  prefers to be matched with  $m_1$  (rather than with  $m_2$ ) then the matching is stable.

If  $m_1$  prefers to be matched with  $w_1$  and  $w_1$  prefers to be matched with  $m_1$  then the matching is unstable and the pair  $(m_1, w_1)$  is a blocking pair.

If  $w_1$  prefers to stay unmatched rather than be matched with  $m_2$ , it is said that  $w_1$  blocks the matching individually. It is important to mention that according to Gayle and Shapley every one-to-one model has stable matching.<sup>16</sup>

The main stability issues remain the same in a many-to-one matching model. The difference is that stability depends not only on individuals and pairs but also on a coalition of agents like students/workers/colleges/firms, and the way that this coalition can block a matching assignment. Roth asserts that "a matching  $\mu$  is blocked by some

coalition A of colleges and students if, by matching among themselves, the students and colleges in A could all get an assignment preferable to  $\mu$ ."<sup>13</sup> In a case that there is no individual, pairwise or coalition blocking, the matching assignment is regarded as stable.<sup>18</sup>

The second important issue is Pareto efficiency. A matching is considered as Pareto efficient (or optimal) for one side of the market when there is no other outcome that makes every player on this side at least as well off and at least one player strictly better off;  $\mu_1$  in example 1 (Figure 8) is a Pareto efficient one. However, a matching  $\mu_i$  ( $m_1$ ,  $w_2$ ), ( $m_2$ ,  $w_3$ ) and ( $m_3$ ,  $w_1$ ) is not Pareto efficient because there is another match that some men ( $m_1$ ) like at least as well as  $\mu_i$  and some men ( $m_2$ ,  $m_3$ ) prefer this than  $\mu_i$ .<sup>17</sup>

Finally a matching mechanism is said to be strategy proof when there is no incentive for any of the agents to lie about or hide their private information from the other agents; that is, the mechanism cannot be manipulated by agents misrepresenting preferences. However, under specific circumstances agents well-informed about other agents' behavior could achieve to be matched with more preferable partners if they lie about their preferences. That is, a non-strategy proof mechanism could be proven better for some individuals. Furthermore, inside an organization the selection and application of a strategy focused on the expression of the other side agents' true preferences would return better results.

Before the author moves forward it would be helpful to mention some further substantial terms of two-sided matching theory:

- An agent has <u>strict preferences</u> when he/she is not indifferent between any two acceptable mates, or between being matched to an acceptable mate and being unmatched.<sup>16</sup>
- A matching is <u>individually rational</u> when each student is assigned to an acceptable school or he/she remains unassigned.<sup>20</sup> In the above examples, the matches are individually rational since all pairs are mutually accepted.
- <u>Justified envy</u> takes place when there is a pair of a student i and a school s he/she was not assigned to, such that i prefers s to his/her assignment, and i has a higher priority than some student who was assigned to s.<sup>21</sup> It is said that matching eliminates justified envy when there is no pair student/school in which the student has the specific school in a lower

priority than another student who was assigned somewhere else.<sup>22</sup> A mechanism eliminates justified envy if it generates a stable matching.<sup>23</sup>

Another referable issue is the optimality. For each marriage model and many-toone model and when preferences are strict, there always exists two optimal stable
matches, men-women optimal and student/worker-college/firm optimal stable matching
respectively. Both optimal stable matches are produced through a specific mechanism
called Deferred Acceptance mechanism (DA), which will be analyzed thoroughly in an
upcoming section. Each of these matches is biased because it is extracted based on the
preferences of each group. For example, in the marriage model a matching is men
optimal if men propose and if every man likes the stable outcome at least the same as any
other stable matching. The women optimal matching is produced in a similar way. The
men optimal matching is the worst case scenario for the women because it matches each
woman with her least preferred partner; that is, it improves men welfare at the expense of
women and vice versa. Nevertheless, in practice the two outcomes are almost the same
and have little difference.<sup>13</sup>

Example No 4	<u>Men</u>	Women	Optimal	Optimal
			matching μ(m)	matching (w)
	$Pm_1(w_2, w_1, w_3)$	$Pw_1(m_3, m_1, m_2)$	$m_1 \sim w_1$	$m_1 \nearrow M_1$
	$Pm_{2}\left(w_{1},w_{3},w_{2}\right)$	$Pw_2 (m_2, m_1, m_3)$	$m_2$ $w_2$	$m_2 \leftrightarrow w_2$
	$Pm_3 (w_1, w_2, w_3)$	$Pw_3 (m_3, m_1, m_2)$	$m_3$ $W_3$	$m_3$ $w_3$

Figure 12. Optimality

#### C. ADVANTAGES - DISADVANTAGES

Two-sided matching is one of many existing ways to face the assignment problem. The advantages of a two-sided matching algorithm's use are the following:

- It takes into consideration the preferences of both parties and tries to balance them. Actually, in some cases the appearance of those preferences does not reflect the real ones as far as potential misrepresentation could be beneficial at least for the agents of one side. However, the application of an appropriate strategy could be proven catalytic to this direction.
- The outcome is stable and it can be an optimal one too.

- When ties appear in the preference list, alternative solutions result and can be explored, analyzed and evaluated by the decision makers.<sup>14</sup>
- Besides and as far as matching markets are concerned, the consideration of both sides' preferences has a positive effect on the supply and demand efficiencies and improves (at least theoretically) the welfare of all involved parties.<sup>24</sup>

Of course, the application of a two-sided matching algorithm has disadvantages too. The major disadvantage is that every agent must fill/submit a complete preference list in a ranked order, including every available position and vice versa in order to avoid the phenomenon that agents/positions will remain unmatched. Moreover, no two-sided matching algorithm can ensure any agent that her top rated priorities will be fulfilled. However, the application of strategies like longer preference lists or secondary matching rounds would be helpful towards that direction.<sup>14</sup>

#### D. APPLYING TWO-SIDED MATCHING

There are many two-sided matching mechanisms that have been applied and are still applied for assignment purposes in organizations and generally in markets of a different nature. The author chooses three mechanisms of them, with different characteristics, advantages and disadvantages to describe and analyze. In the next chapter, and after the author comprehends the function of these mechanisms, he applies them in order to assign the Hellenic Navy officers to positions. Finally, evaluation and comparison of the three mechanisms' results will take place in order to assist the Hellenic Navy's relevant office in assignment procedure.

#### 1. Priority Mechanism

The Priority mechanism was applied in the UK concerning the assignment of students to particular hospital programs. Under this mechanism, a priority to each match is assigned based on the submitted preference rankings of sides A (consultant) and B (student). According to Unver, when a student i lists a consultant s in the kth spot in his/her rank order list and the same consultant lists the student in lth spot, such a (s,i) match is called a (k,l) match. The priority of a (k, l) match is the product of the student's ranking of the consultant and consultant's ranking of the student, that is k x  $1^{25}$ .

Therefore, in a (1,1) match both sides ranked each other first and the priority number is 1 (1 x 1); similarly, the matches (1,2) and (2,1) have the same priority, which is 2 and so on. After all priorities are assigned, the matches are generated starting from the lowest priority number.

Two versions of the priority mechanism appeared differing in the way that they broke ties; the first version (applied in Birmingham) broke ties in the consultant's favor (that is a (1,2) match had a higher priority than a (2,1) match) while the second version (applied in Newcastle) broke ties in the student's favor (that is a (2,1) match had a higher priority than a (1,2) match).<sup>13</sup>

Both versions of the Priority mechanism are unstable, which means there may exist a pair (consultant-student in this case) that prefers each other instead of their generated partner. Furthermore, they are neither Pareto efficient nor usually strategy proof. These negative features in combination with "prior arrangement phenomenon" that appeared led to failure and the abandonment of the specific mechanism even though (1,1) matches were always realized.

#### 2. Deferred Acceptance Mechanism (DA)

The Deferred Acceptance mechanism was proposed by Gale and Shapley (1962) using marriage partners for illustration purposes<sup>26</sup> and its title emphasizes the technique that is followed throughout its application. It was adopted and applied (in its first and amended versions) in various labor markets including the education market (school choice/college admission).

Under the specific mechanism, the agents on each side make proposals to the agents of the other side according to their preferences. Agents that receive more proposals than they can accept keep (but not engages to) the most preferable and reject the others. Rejected agents propose again and the new proposals are evaluated; some of them are kept and some of them are rejected again including proposals that were held in the previous steps, but they are less preferable than the new ones (that means that acceptances are deferred throughout the mechanism and until there are no further

proposals or until the mechanism's end). The mechanism is terminated when the rejected agents cannot make more proposals and the kept proposals at this step are considered as the final ones.<sup>16</sup>

Assuming that there are two groups of agents, A and B, the question is: which group's agents propose and which group's agents keep, accept or reject the proposals? The answer is that for every given market and when the agents of both groups have strict preferences, there exists two optimal stable matches, one based on the proposals of the first group and another one based on the proposals of the second group. When the agents of group A propose, the result is A-optimal stable matching (each A's agent likes the matching at least as well as any other stable matching) and when the agents of group B propose the result is B-optimal stable matching (each B's agent likes the matching at least as well as any other stable matching). Also, the author has to mention that A-optimal stable matching is the worst case scenario for B's agents and vice versa.

The Deferred Acceptance mechanism produces stable matching, which means there is no unmatched pair (c, d) where agent c prefers agent d instead of his/her assigned partner and he/she has higher priority from the agent (or the agents) of his/her group who are assigned to agent d. Furthermore, the mechanism is a strategy-proof one that eliminates justified envy. It is important to indicate that the complete elimination of justified envy may be in conflict with Pareto efficiency while a potential trade-off between stability and Pareto efficiency may appear.<sup>17</sup>

#### **3.** Top Trading Cycles (TTC) Mechanism

The TTC mechanism's version that the author examines is introduced by Abdulkadiroglu & Sonmez (2003) and is a competing mechanism to the Deferred Acceptance algorithm that refers to the school choice/college admission problem. It starts with students who have the highest priorities and allows them to trade the schools for which they have the highest priorities. Once these students are removed, it proceeds in a similar way starting with students who have the highest priorities among those who remain. As Abdulkadiroglu and Sonmez explain, the mechanism works as follows:<sup>17</sup>

Step 1: A counter is assigned for each school  $(s_i)$  that marks the available seats of the school (initially equal to the capacity of school). Each student  $(i_i)$  points to his/her favorite school according to his/her preferences and each school points to the student who is first in the school's priority order. After that, at least one cycle appears, that is an ordered list of schools and students; i.e.,  $(s_1, i_3, s_2, i_5)$ . In this case, student  $i_3$  is assigned to school  $s_2$  and student  $i_5$  is assigned to school  $s_1$ . Each agent (school or student) can be part of one cycle at most. Each student that is assigned to a school is removed, and similarly each school in which a student is assigned is removed too unless there are more than one seat available; in that case, the counter is reduced by one and the school remains active in the procedure. All other schools' counters remain the same.

Step k: Each remaining student points to his/her favorite school among the remaining schools and each school points to the student who is first in the school's priority order among the remaining students. The procedure is the same as step 1. The mechanism terminates whenever all students are assigned to a school or all available school seats are covered by students.

The TTC mechanism, as it is performed by Abdulkadiroglu and Sonmez, seems to function in favor of students; that means when a cycle appears the students are assigned to the school that they pointed to. For example,  $i_3$  is assigned to his/her school of preference  $s_1$  and  $i_5$  is assigned to  $s_2$ . In the next chapter's example the author is going to check the function of the mechanism in favor of schools' preferences too.

The TTC mechanism is Pareto efficient ("because no student can be better off without hurting someone who left the mechanism in a previous step"<sup>17</sup>) and a strategy proof mechanism<sup>23</sup>; that is, the declaration of students' true preferences is a dominant strategy for them. This happens because in each step the mechanism is based on the highest priorities and in a case of misrepresenting preferences the student's true preference will leave the mechanism in a previous step. Thus, a potential manipulation will have a negative effect for the student.<sup>17</sup> On the other hand the application of the TTC mechanism cannot eliminate justified envy completely.

A visual image concerning the three above described two-sided matching mechanisms and their attributes is shown in Table 2.

Table 2. Two-Sided Matching Mechanisms and their Attributes

Attributes/ Mechanisms	Priority	Gale- Shapley Deferred Acceptance	Top Trading Cycle
Stability	No	Yes	No
Strategy proof	No	Yes	Yes
Pareto Efficient	No	No*	Yes
Complete elimination of	No	Yes*	No
justified envy			
*: in conflict			

# IV. APPLICATION OF TWO SIDED-MATCHING MECHANISMS IN THE HELLENIC NAVY ASSIGNMENT PROCEDURE

#### A. ANALYSIS OF THE MECHANISMS

The author assumes that there are ten positions in the Hellenic Navy (P1, P2, ..., P10) that have to be covered, and fifteen officers (O1, O2, ..., O15) are eligible (according to their rank) for the specific assignments. Each position has to be covered by one officer except P1, which requires two officers. Each officer has filled the matrix with his annual preferences; an overall view with all officers' preferences appears in the following Table 3.

Table 3. Officers' Preference List

	$1^{st}$	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	$2^{nd}$	$3^{\text{rd}}$	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
O8:	P9	P8	P4	P6	P7						

In this specific example and for the creation of this matrix, the positions are divided into groups of similar nature (like "off-center" positions, abroad or not, commander positions, etc.). The officers' preferences are based on this division; that means an officer who desires to be placed as a commander has as first preferences the relative positions, the same happens with an officer who desires to be placed abroad or in a domestic "off-center" position, etc.

Furthermore, another matrix (Table 4) that indicates each position's "preferences" based on the eligible officers is needed. This matrix is created (as explained in Chapter II) by taking into consideration the characteristics of each officer (namely, the possession of

a master's degree, the numerical value of his/her experience, and the average of his/her evaluation reports) and the specific requirements that each position has. As an example, for some positions the order of priorities is experience/possession of a master's degree/evaluation reports while for other positions the order is different according to the nature of the position.

Table 4. Positions' Preference List

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	06	015	011	O4	01	09	03	O2	05	O7	013	014	O12	O8	010
P2:	O4	06	011	O15	O7	01	O3	09	O2	O5	014	O12	08	O10	013
P3:	O4	06	01	O5	011	015	09	O7	O3	012	O2	010	08	013	014
P4:	O10	08	013	O12	01	011	O4	O15	06	09	014	O7	O2	O5	03
P5:	O12	08	01	O10	013	06	04	09	011	015	O2	O7	O5	O3	014
P6:	O10	O12	08	013	01	09	06	015	011	O4	014	O2	O3	O7	O5
P7:	O3	O5	09	O7	02	014	010	013	08	012	06	O4	015	011	01
P8:	06	O4	011	01	015	O7	O10	O13	O12	08	014	O5	09	O3	O2
P9:	O10	O12	013	08	02	O5	014	O7	09	011	04	015	01	O3	06
P10:	O3	O5	O2	O7	09	012	08	014	013	010	015	011	06	01	O4

In this specific example, the officers are divided into groups according to the above mentioned characteristics. Consequently, according to the position's order of priorities the respective group of officers is chosen (i.e., if the nature of the position requires an order like possession of a master's degree/experience/evaluation reports then the group of officers with a master's degree will be chosen first, the group of officers with the higher level of experience will be chosen second and the group of officers with higher average in the evaluation reports will be chosen last). In cases where more officers than needed cover the first criterion, the second criterion (and if needed the third criterion) will clarify who is going to match to the specific positions. Thus, the criteria are not mutually exclusive and an officer may qualify for two or even all of them simultaneously.

It needs to be clarified that in most cases the connection between a position's requirements and its order of priorities according to the above mentioned characteristics is likely to be subjective; thus, the staff of the Department of Personnel has to take into account the parameters in order to create the matrix with positions' "preferences."

#### In this particular example:

- For P1, P2, and P3 the priority is evaluation reports/possession of a master's degree/ experience.
- For P4, P5, and P6 the priority is experience, evaluation reports/possession of a master's degree.
- For P7 and P10 the priority is possession of a master's degree/ experience/ evaluation reports.
- For P8 the priority is evaluation reports/experience/possession of a master's degree.
- For P9 the priority is experience/possession of a master's degree/ evaluation reports.
- The officers that have a relative advantage in each of the positions according to the priorities are shown in Table 4; i.e., for P1, P2 and P3 positions the officers that have the advantage to cover them are O4, O6, O11, O15, O1, etc.

Having the data of the above tables, the author will apply the three different twosided matching mechanisms that are described in the previous chapter in order to match the Hellenic Navy positions to the Greek officers eligible for these positions.

## 1. Matching Mechanism Number 1: Priority

Potential match	O's position in Q(P)		P's position in Q(O)		Priority number	Potential match	O's position in Q(P)		P's position in Q(O)		Priority number
P1, O1	5	X	5	=	25	P2, O1	6	X	2	=	12
P1, O2	8	X	-	=	-	P2, O2	9	X	-	=	_
P1, O3	7	X	4	=	28	P2, O3	7	X	2	Ш	14
P1, O4	4	X	2	=	8	P2, O4	1	X	1	Ш	1
P1, O5	9	X	3	=	27	P2, O5	10	X	-	=	-
P1, O6	1	X	4	=	4	P2, O6	2	X	-	=	-
P1, O7	10	X	3	=	30	P2, O7	5	X	1	=	5
P1, O8	14	X	-	=	-	P2, O8	13	X	-	=	-
P1, O9	6	X	4	=	24	P2, O9	8	X	1	=	8
P1, O10	15	X	3	=	45	P2, O10	14	X	2	=	28
P1, O11	3	X	2	=	6	P2, O11	3	X	1	=	3
P1, O12	13	X	1	=	13	P2, O12	12	X	-	=	-
P1, O13	11	X	-	=	-	P2, O13	15	X	2	=	30
P1, O14	12	X	3	=	36	P2, O14	11	X	2	=	22
P1, O15	2	X	5	=	10	P2, O15	4	X	-	=	-

Potential match	O's position in Q(P)		P's position in Q(O)		Priority number	Potential match	O's position in Q(P)		P's position in Q(O)		Priority number
P3, O1	3	X	1	=	3	P4, O1	5	X	_		_
P3, O2	9	X	-	=	-	P4, O2	13	X	1	Ш	13
P3, O3	12	X	1	=	12	P4, O3	15	X	-	Ш	-
P3, O4	1	X	-	=	-	P4, O4	7	X	3	Ш	21
P3, O5	4	X	-	=	4	P4, O5	14	X	2	Ш	28
P3, O6	2	X	-	=	-	P4, O6	9	X	2	Ш	18
P3, O7	7	X	-	=	-	P4, O7	12	X	4	Ш	48
P3, O8	11	X	-	=	-	P4, O8	2	X	3	Ш	6
P3, O9	6	X	-	=	-	P4, O9	10	X	-	Ш	-
P3, O10	10	X	1	=	10	P4, O10	1	X	4	Ш	4
P3, O11	14	X	-	=	-	P4, O11	6	X	-	=	-
P3, O12	8	X	-	=	-	P4, O12	4	X	-	=	-
P3, O13	12	X	1	=	12	P4, O13	3	X	4	Ш	12
P3, O14	15	X	1	=	15	P4, O14	11	X	-	Ш	-
P3, O15	6	X	-	=	-	P4, O15	8	X	2	=	16

Potential match	O's position in Q(P)		P's position in Q(O)		Priority number	Potential match	O's position in Q(P)		P's position in Q(O)		Priority number
P5, O1	3	X	3	=	9	P6, O1	5	X	-	=	_
P5, O2	11	X	-	=	1	P6, O2	12	X	3	Ш	36
P5, O3	14	X	-	=	1	P6, O3	13	X	-	Ш	-
P5, O4	7	X	-	=	ı	P6, O4	10	X	5	Ш	50
P5, O5	13	X	-	=	1	P6, O5	15	X	-	=	-
P5, O6	6	X	5	=	30	P6, O6	7	X	3	=	21
P5, O7	12	X	2	=	24	P6, O7	14	X	5	=	70
P5, O8	12	X	-	=	-	P6, O8	3	X	4	=	12
P5, O9	8	X	-	=	1	P6, O9	6	X	5	=	30
P5, O10	4	X	-	=	-	P6, O10	1	X	-	=	-
P5, O11	9	X	-	=	-	P6, O11	9	X	5	=	45
P5, O12	1	X	5	=	5	P6, O12	2	X	-	=	-
P5, O13	5	X	-	=	-	P6, O13	4	X	-	=	-
P5, O14	15	X	5	=	75	P6, O14	11	X	4	=	44
P5, O15	10	X	-	=	-	P6, O15	8	X	4	=	32

Potential match	O's position in Q(P)		P's position in Q(O)		Priority number	Potential match	O's position in Q(P)		P's position in Q(O)		Priority number
P7, O1	15	X	-	=	-	P8, O1	4	X	-	=	-
P7, O2	5	X	4	=	20	P8, O2	15	X	2	Ш	30
P7, O3	1	X	5	=	5	P8, O3	14	X	3	Ш	42
P7, O4	12	X	-	=	-	P8, O4	2	X	-	=	-
P7, O5	2	X	4	=	8	P8, O5	12	X	1	=	12
P7, O6	11	X	-	=	-	P8, O6	1	X	1	=	1
P7, O7	4	X	-	=	-	P8, O7	6	X	-	=	-
P7, O8	9	X	5	=	45	P8, O8	10	X	2	=	20
P7, O9	3	X	-	=	-	P8, O9	13	X	3	Ш	39
P7, O10	7	X	-	=	-	P8, O10	7	X	-		-
P7, O11	14	X	4	=	56	P8, O11	3	X	-	Ш	-
P7, O12	10	X	4	=	-	P8, O12	9	X	2	=	18
P7, O13	8	X	5	=	40	P8, O13	8	X	-	Ш	_
P7, O14	6	X	-	=	-	P8, O14	11	X	-	Ш	_
P7, O15	13	X	3	=	39	P8, O15	5	X	-	Ш	_

Potential match	O's position in Q(P)		P's position in Q(O)		Priority number	Potential match	O's position in Q(P)		P's position in Q(O)		Priority number
P9, O1	13	X	4	=	52	P10, O1	14	X	-	=	-
P9, O2	5	X	-	=	-	P10, O2	3	X	5	Ш	15
P9, O3	14	X	-	=	-	P10, O3	1	X	-	Ш	-
P9, O4	11	X	-	=	-	P10, O4	15	X	4	=	60
P9, O5	6	X	-	=	-	P10, O5	2	X	5	=	10
P9, O6	15	X	-	=	-	P10, O6	13	X	-	=	-
P9, O7	8	X	-	=	-	P10, O7	4	X	-	Ш	-
P9, O8	4	X	1	=	4	P10, O8	7	X	-	Ш	-
P9, O9	9	X	2	=	18	P10, O9	5	X	-	Ш	-
P9, O10	1	X	5	=	5	P10, O10	10	X	5	Ш	50
P9, O11	10	X	-	=	-	P10, O11	12	X	3	Ш	36
P9, O12	2	X	-	=	-	P10, O12	6	X	3	=	18
P9, O13	3	X	3	=	9	P10, O13	9	X	5	Ш	45
P9, O14	7	X	-	=	-	P10, O14	8	X	-	Ш	_
P9, O15	12	X	1	=	12	P10, O15	11	X	5	Ш	55

Before the author starts the analysis it is important to mention that in the case of a tie, the author breaks it in favor of the positions' "preferences."

<u>Step 1:</u> Taking into account the lowest priority number of each potential match, the ranked ordered lists results in the following matches:

Step2: P2, P3, P4, P5, P7 and P10 have no immediate competitors. Unlikely, P1 and P8 have the same first choice; P1 has to concede its first choice to P8 and proposes to second-ranked O11. Similarly P6 has to concede its first choice to P9 and propose to its second choice, O6.

Step 3: P6 proposes to third-ranked O9 because O6 has already been matched. As far as P1 is concerned, O11 covers the first of the two available positions. The fourth-ranked O15 is proposed for the second available position because the previous ranked options have already been matched.

Step 4: P1 matches to O15. P6 proposes and matches to the third-ranked O9.

The final ranked ordered list resulted in the following matches:

Table 5. Priority Mechanism's Final Matching

Potential Match	O's position in Q(P)		P's position in Q(O)		Priority Number
P2, O4	1	X	1		1
P8, O6	1	X	1	Ш	1
P3, O1	3	X	1	=	3
P4, O10	1	X	4	=	4
P9, O8	4	X	1	=	4
P5, O12	1	X	5		5
P7, O3	1	X	5		5
P1, O11	3	X	2	Ш	6
P10, O5	2	X	5		10
P1, O15	2	X	5		10
P6, O9	6	X	5	=	30

#### 2. Matching Mechanism Number 2: Deferred Acceptance (DA)

• Matching based on officers' preferences

Step 1: O2 and O12 have no immediate competitors, so they engage to P4 and P1 respectively. Unlikely O1, O3, O10, O13 and O14 propose to P3; O4, O7, O9 and O11 propose to P2; O5 and O6 propose to P8; O8 and O15 propose to P9. P3 rejects O3, O10, O13 and O14 and keeps O1 engaged; P2 rejects O7, O9 and O11 and keeps O4 engaged; P8 rejects O5 and keeps O6 engaged; P9 rejects O15 and keeps O8 engaged.

The author indicates this in the following manner:

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
O12	O4	O1	O2				O6	08	

Step 2: O3, O5, O7, O9, O10, O11, O13, O14 and O15 propose to their second choice, namely to P2, P4, P5, P9, P2, P1, P2, P2 and P4 respectively. P1 keeps O11 engaged for the second available position; P2 rejects O3, O10, O13, and O14 and keeps O4 engaged; P4 rejects O2, O5 and keeps O15 engaged; P5 keeps O7 engaged; P9 rejects O9 and keeps O8 engaged.

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
O11, O12	O4	O1	O15	O7			O6	O8	

Step 3: O2 propose to its second choice (P8) while O3, O5, O9, O10, O13 and O14 propose to their third choice, namely to P8, P1, P8, P1, P9 and P1 respectively. P1 rejects O10,O12 and O14 and keeps O5,O11 engaged; P8 rejects O2, O3, O9, and keeps O6 engaged; P9 rejects O8 and keeps O13 engaged.

Step 4: O8 and O12 propose to their second choice (P8 both) while O2 proposes to its third choice (P6) and O3, O9, O10, and O14 propose to their fourth choice (P1, P1, P4, and P6 respectively). P1 rejects O3, O5 and keeps O9, O11 engaged; P4

rejects O15 and keeps O10 engaged; P6 rejects O2 and keeps O14 engaged; P8 rejects O8, O12 and keeps O6 engaged.

Step 5: O8, O12, O15 propose to their third choice (P4, P10 and P7 respectively) while O2, O5 propose to their fourth choice (P7 both) and O3 proposes to its fifth choice (P7). P4 rejects O8 and keeps O10 engaged; P7 rejects O2, O5 and O15 and keeps O3 engaged. P10 is engaged to O12.

Step 6: O8, O15 propose to their fourth choice (P6 both) while O2, O5 and O14 propose to their fifth choice (P10, P10 and P5 respectively). P5 rejects O14 and keeps O7 engaged; P6 rejects O15 and keeps O8 engaged; P10 rejects O2, O12 and keeps O5 engaged.

Step 7: O12 proposes to its fourth choice (P7) while O15 proposes to its fifth choice, namely P1. P7 rejects O12 and keeps O3; P1 rejects O9 and keeps O11, O15 engaged.

Step 8: O9 proposes to its fifth choice, namely P6. P6 rejects O9 and keeps O8 engaged. O12 proposes to its fifth choice, that is P5. P5 rejects O7 and keeps O12 engaged.

Step 9: O7 proposes to its third choice (P1) but P1 rejects O7 and keeps O11, O15 engaged. After that, O7 proposes to its fourth choice (P4) but P4 rejects O7 and keeps O10 engaged. Finally, O7 proposes to its fifth choice (P6) but P6 rejects O7 and keeps O8 engaged.

Consequently, the final matching  $\mu(O)$ , which is the matching resulting from the procedure driven by the officers' preferences, is the following:

Table 6. Deferred Acceptance Based on Officers' Preferences Final Matching

P1	P2	Р3	P4	P5	Р6	P7	Р8	Р9	P10
011, 015	04	01	010	012	08	03	06	013	<b>O</b> 5

Furthermore, a visual image of the procedure is as follows.

Table 7. Deferred Acceptance Based on Officers' Preferences Example- Step 1

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
O8:	P9	P8	P4	P6	P7						

Table 8. Deferred Acceptance Based on Officers' Preferences Example- Step 2

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
07	DO	D.5	D1	D.4	De	015	DO.	D.4	D.7	De	D1
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
00	DO	DO	D4	De	D7						
O8:	P9	P8	P4	P6	P7						

Table 9. Deferred Acceptance Based on Officers' Preferences Example- Step 3

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
O8:	P9	P8	P4	P6	P7						

Table 10. Deferred Acceptance Based on Officers' Preferences Example- Step 4

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
0.	7.0	5-	5.4	5.1		01.5	D.O.		<b>5-</b>	<b>D</b> =	- D4
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
O8:	P9	P8	P4	P6	P7						

Table 11. Deferred Acceptance Based on Officers' Preferences Example- Step 5

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
						0.1.0		20	7.0		
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
0.5	DO	D.1	D1	D.7	D10	010	DO.	Da	Do	D.4	D10
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
06	Do	D4	DC	D1	D6	014	D2	D2	D1	DC	D5
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
07.	ΓZ	ГЭ	L 1	Г4	FU	015.	ГЭ	Γ4	Г/	ΓU	Г1
O8:	P9	P8	P4	P6	P7						

Table 12. Deferred Acceptance Based on Officers' Preferences Example- Step 6

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
0.5		5-	54		<b>D</b>	04.5	Do		25	D.	
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
0.0	<b>D</b> 0	700	<b>D</b> 4	D.(	<b>5</b> -						
O8:	P9	P8	P4	P6	P7						

Table 13. Deferred Acceptance Based on Officers' Preferences Example- Step 7

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
		7.0			7.10	0.10			-		
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
03.	13	12	10	11	1 /	011.	1 2	1 1	110	1 /	10
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
							7.0				
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
00	DO	DO	D4	DC	D7						
O8:	P9	P8	P4	P6	P7						

Table 14. Deferred Acceptance Based on Officers' Preferences Example- Step 8

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
0.1			<b>D</b> -			044	- DO	7.0	7.1		7.
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
07	DO.	D.C	D1	D4	De	015	DO	D4	D7	DC	D1
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
00	DO	DO	D4	De	D7						
O8:	P9	P8	P4	P6	P7						

Table 15. Deferred Acceptance Based on Officers' Preferences Example- Step 9

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	P2	P9	P8	P1	P6
O2:	P4	P8	P6	P7	P10	O10:	P3	P2	P1	P4	P9
O3:	P3	P2	P8	P1	P7	O11:	P2	P1	P10	P7	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P8	P10	P7	P5
O5:	P8	P4	P1	P7	P10	O13:	P3	P2	P9	P4	P10
O6:	P8	P4	P6	P1	P5	O14:	P3	P2	P1	P6	P5
O7:	P2	P5	P1	P4	P6	O15:	P9	P4	P7	P6	P1
O8:	P9	P8	P4	P6	P7						

#### • Matching based on positions' requirements

Step 1: P5 has no immediate competitors, so it engages to O12. Unlikely P1 and P8 propose to O6; P2 and P3 propose to O4; P4, P6 and P9 propose to O10; P7 and P10 propose to O3. O3 rejects P10 and keeps P7 engaged; O4 rejects P3 and keeps P2 engaged; O6 rejects P1 and keeps P8 engaged; O10 rejects P6, P9 and keeps P4 engaged.

The author indicates this in the following manner:

01	O2	O3	O4	O5	06	O7	08	O9	O10	O11	O12	O13	O14	O15
-	-	P7	P2	ı	P8	ı	1	ı	P4	ı	P5	ı	-	-

Step 2: P1, P3, P6, P9 and P10 propose to their second choice, namely to O15, O6, O12, O12 and O5 respectively. O15 keeps P1 engaged; O5 keeps P10 engaged; O6 rejects P3 and keeps P8 engaged; O12 rejects P6, P9 and keeps P5 engaged.

01	O2	O3	O4	O5	06	O7	O8	O9	O10	O11	O12	O13	O14	O15
-	-	P7	P2	P10	P8	-	-	-	P4	-	P5	-	-	P1

Step 3: P3, P6 and P9 propose to their third choice, that is, to O1, O8 and O13 respectively. O1 keeps P3 engaged; O8 keeps P6 engaged; O13 keeps P9 engaged. Furthermore, P1 keeps engaged to O15 but also proposes to its third choice, which is O11, because it has one more position available.

Consequently, the final matching  $\mu(P)$ , which is the matching resulting from the procedure driven by the positions' requirements, is the following:

Table 16. Deferred Acceptance Based on Positions' Preferences Final Matching

01	O2	03	04	05	06	07	08	09	O10	011	O12	013	014	015
P3	-	P7	P2	P10	P8	-	P6	-	P4	P1	P5	P9	-	P1

Furthermore, a visual image of the procedure is as follows.

Table 17. Deferred Acceptance Based on Positions' Preferences Example- Step 1

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	06	O15	O11	O4	01	09	O3	O2	O5	O7	O13	O14	O12	08	O10
P2:	O4	O6	O11	O15	O7	01	O3	09	O2	O5	O14	O12	08	O10	O13
P3:	O4	O6	O1	O5	011	O15	O9	O7	O3	O12	O2	O10	08	O13	O14
P4:	O10	O8	O13	O12	01	011	O4	O15	06	09	O14	O7	O2	O5	O3
P5:	O12	O8	O1	O10	O13	06	O4	09	011	015	O2	O7	O5	O3	O14
P6:	O10	O12	O8	O13	01	09	06	O15	011	04	O14	O2	03	O7	O5
P7:	O3	O5	O9	O7	O2	O14	O10	O13	08	O12	06	O4	015	011	O1
P8:	06	O4	011	O1	015	O7	O10	O13	O12	08	O14	05	09	O3	O2
P9:	O10	O12	O13	O8	02	O5	O14	O7	09	011	O4	015	01	O3	06
P10:	O3	O5	O2	O7	09	O12	08	O14	O13	O10	015	011	06	01	O4

Table 18. Deferred Acceptance Based on Positions' Preferences Example- Step 2

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	06	O15	011	O4	01	09	O3	O2	O5	O7	O13	O14	O12	08	O10
P2:	O4	06	O11	015	O7	01	O3	09	O2	O5	O14	O12	08	O10	013
P3:	O4	06	01	O5	011	015	09	O7	O3	O12	O2	O10	08	013	014
P4:	O10	O8	O13	O12	01	011	O4	O15	06	09	O14	O7	O2	O5	O3
P5:	O12	O8	01	O10	013	06	O4	09	011	015	O2	O7	O5	O3	O14
P6:	O10	O12	08	O13	01	09	06	015	011	04	O14	O2	O3	O7	O5
P7:	O3	O5	09	O7	02	014	O10	013	08	O12	06	04	015	011	01
P8:	06	O4	011	01	015	O7	O10	013	O12	08	O14	05	09	O3	O2
P9:	O10	O12	O13	O8	02	O5	O14	O7	09	011	04	015	01	O3	06
P10:	O3	O5	O2	O7	09	O12	08	O14	013	O10	O15	011	06	01	O4

Table 19. Deferred Acceptance Based on Positions' Preferences Example- Step 3

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	06	O15	011	O4	01	09	O3	O2	O5	O7	O13	O14	O12	08	O10
P2:	O4	O6	011	O15	O7	01	O3	09	O2	O5	O14	O12	08	010	013
P3:	O4	O6	01	O5	011	015	09	O7	O3	O12	O2	O10	08	013	014
P4:	O10	O8	O13	O12	01	011	O4	O15	06	09	O14	O7	O2	O5	O3
P5:	O12	O8	01	O10	013	06	O4	09	011	015	O2	O7	05	O3	O14
P6:	O10	O12	O8	O13	01	09	06	015	011	04	O14	O2	03	O7	O5
P7:	O3	O5	09	O7	O2	014	O10	O13	08	O12	06	04	015	011	01
P8:	06	O4	011	01	015	O7	O10	013	O12	08	O14	05	09	O3	O2
P9:	O10	O12	O13	O8	O2	O5	O14	O7	09	011	O4	015	01	O3	06
P10:	O3	O5	O2	O7	09	012	08	O14	013	O10	015	011	06	O1	O4

### 3. Matching Mechanism Number 3: Top Trading Cycle mechanism

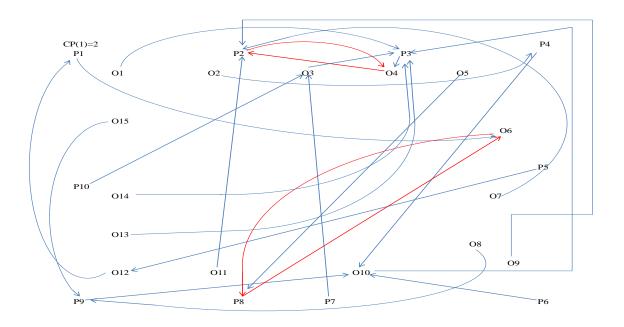


Figure 13. Top Trading Cycle Example - Step 1

There are two cycles in Step 1: (P2, O4) and (P8, O6). Therefore, officers O4 and O6 are assigned to positions P2 and P8 respectively and removed. In addition, positions P2 and P8 are removed.

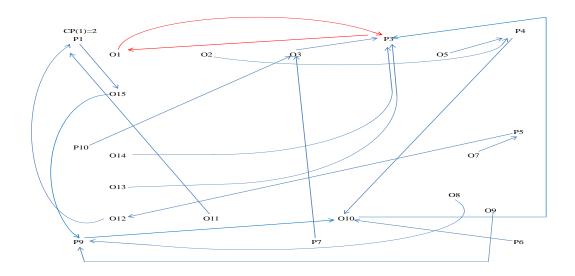


Figure 14. Top Trading Cycle Example - Step 2

There is one cycle in Step 2, which is (P3, O1). Therefore, officer O1 is assigned to position P3 and removed. In addition, position P3 is removed.

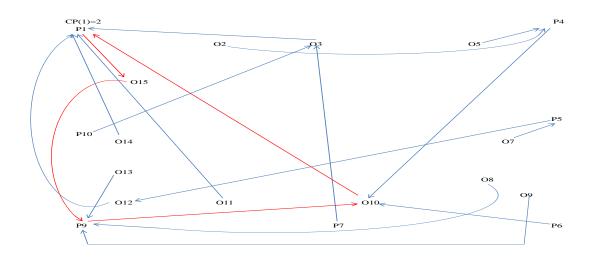


Figure 15. Top Trading Cycle Example - Step 3

There is one cycle in Step 3, which is (P1, O15, P9, and O10). Therefore, according to the mechanism officers O10 and O15 are assigned to positions P1 and P9 respectively and removed. In addition, position P9 is removed while position P1 is reduced by one for the next step. However, what happens if the assignments take place in favor of schools? In such a case the matching would be (P1, O15) and (P9, O10).

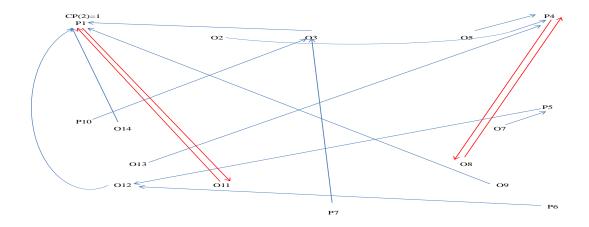


Figure 16. Top Trading Cycle Example - Step 4

There are two cycles in Step 4: (P1, O11) and (P4, O8). Therefore, officers O11 and O8 are assigned to positions P1 and P4 respectively and removed. In addition, positions P1 and P4 are removed.

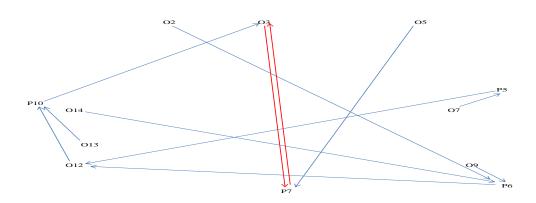


Figure 17. Top Trading Cycle Example - Step 5

There is one cycle in Step 5, which is (P7, O3). Therefore, officer O3 is assigned to position P7 and removed. In addition, position P7 is removed.

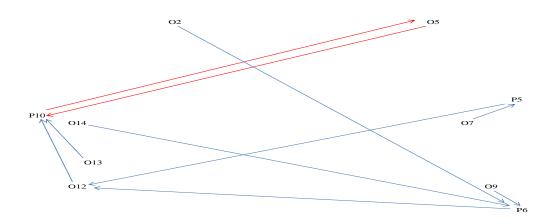


Figure 18. Top Trading Cycle Example - Step 6

There is one cycle in Step 6, which is (P10, O5). Therefore, officer O5 is assigned to position P10 and removed. In addition, position P10 is removed.

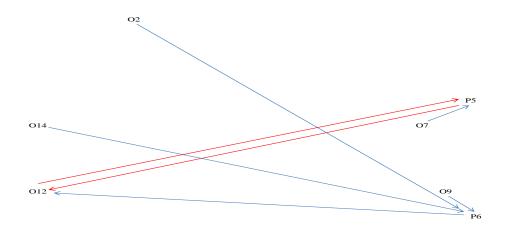


Figure 19. Top Trading Cycle Example - Step 7

There is one cycle in Step 7, which is (P5, O12). Therefore, officer O12 is assigned to position P5 and removed. In addition, position P5 is removed.

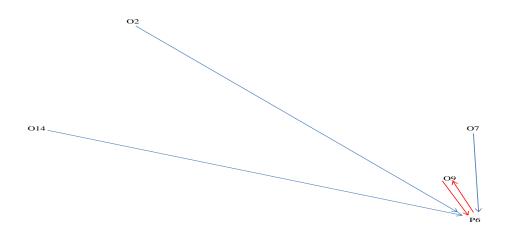


Figure 20. Top Trading Cycle Example - Step 8

There is one cycle in Step 2, which is (P6, O9). Therefore, officer O9 is assigned to position P6 and removed. In addition, position P6 is removed.

There are no remaining positions and the algorithm is terminated here. The final matching is the following:

		Table	20. Т	Top Trad	ling Cy	cle Fina	al Match	ning		
P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	
O10, O11	O4	O1	O8	O12	O9	О3	O6	O15	O5	

By using the amended version of the Abdulkadiroglu and Sonmez (2003) TTC mechanism, functioning in favor of positions the final matching is the following:

Table 21. Top Trading Cycle in Favor of Positions' Final Matching

#### B. RESULTS

Three different matching mechanisms are applied in order to match fifteen Greek officers to ten specific positions of the Hellenic Navy. Before analyzing the results it would be helpful to highlight some interesting points coming from the two-sided matching theory due to the above mentioned application in combination with the specific nature of the military environment:

#### 1. Highlighted Points

- The application of a two-sided matching model in the Hellenic Navy can be a one-to-one model (i.e., commanders or commanding/executive officers' assignments) or a many -to-one model (i.e., assignment of superior officers in ships). In the specific example the author chooses a simple many-to-one model with few one-to-one positions and one position covered by more than one officer.
- Both officers and positions have strict preferences.
- It is not possible to evaluate the rationality of the matching. The reason is that the officers' preference lists allow them to complete just few acceptable positions so it cannot be said for sure if they are finally assigned to one of them. Nevertheless, the option for an officer to remain unassigned in the case of unacceptable matching does not exist, thus he/she has to accept it whether he/she is happy or not.
- In my opinion every two-sided matching model applied in the Hellenic Navy assignment procedure seems to be strategy proof, which means that officers' and positions' preference lists contain true and not misrepresenting information. That is definitely logical for the positions' side. The reason that takes place in the officers' side is because in case of misrepresenting information, the true preferable matching of an officer would leave the model in a previous step and not give him/her the option to be its mate. From my experience that is the rule, but exceptions can also appear; an officer who is well informed about other officers' behavior and preferences would maybe prefer not to state truly the order of his/her

preferences because he/she thinks that for some reasons he/she will not be assigned to one of them. Thus, the author accepts the strategy proof in the Hellenic Navy's assignment process as "anecdotal evidence" by the time that this has not been scientifically measured or proved nor studied.

• Justified envy can be completely eliminated just in the Deferred Acceptance mechanism based on positions' preferences. But we have to mention that, the attempt to eliminate justified envy completely might affect (trade off) the stability and the optimality of the mechanism.

# 2. Comparison of the Mechanisms' Results

The comparison of the three mechanisms' final results shows that:

- The three mechanisms have six common pairs out of eleven, namely (P2, O4), (P3, O1), (P5, O12), (P7, O3), (P8, O6) and (P10, O5).
- Just two of the common pairs {(P2, O4), (P8, O6)} are (1,1).
- Priority mechanism's final matching is unstable because there are pairs of agents, i.e., (P1, O4), (P6, O6), that have not been assigned to each other even though they preferred it rather than their match, and thus it is said that they block the mechanism. The instability means that positions that require specific criteria covered by officers not qualified enough (i.e., P6 requires great experience but it is covered by its 8<sup>th</sup> choice, O15, namely by an officer without it or with less experience than needed).
- The two versions of the Deferred Acceptance mechanism have identical results. That means that each position is covered by exactly the same officer; even the two officers that cover P1 are exactly the same. Furthermore, the final matching is stable; there are no blocking pairs but just individual blocking agents (absolutely logical in such a great example). That means that priorities are followed and positions are covered by qualified officers.
- P1's both available positions are matched to officers by applying all the type of mechanisms. O11 is the first officer who covers P1 in all mechanisms while the second officer is O15 in four cases and O10 in one case (TTC).
- The Deferred Acceptance mechanism's final pairs are more favorable to the positions' preferences than the officers' preferences. In five pairs the officer was the (respective) position's first choice, in two pairs the position's second choice, and in four pairs the position's third choice. Reversely, in three pairs the position was the officer's first choice, in one pair the officer's second choice, in one pair the officer's third choice, in two pairs the officer's fourth choice, and in four pairs the officer's fifth choice. As far as the priority and the Top Trading Cycle mechanisms are

- concerned their final pairs seem more balanced between positions' and officers' preferences than the Deferred Acceptance final pairs.
- As far as Top Trading Cycle mechanism is concerned the final matching is unstable for both versions wit blocking pairs. This instability means that positions that require specific criteria are covered by not qualified officers.
- In general, from the positions' perspective the final pairs extracted from the Deferred Acceptance mechanism are likely to be more favorable, while from the officers' perspective the results extracted from the three mechanisms are similar and the differences insignificant.

#### C. FEW MORE EXAMPLES

So far, the author has tried to describe the application of various two-sided matching mechanisms in a usual assignment problem, concerning ten positions and fifteen officers of the Hellenic Navy according to each side's preferences. Important conclusions were extracted and the comparison of the mechanisms' results revealed the advantages and disadvantages of each one. Subsequently, the author is going to extend his analysis by providing a few examples that will try to cover possible situations that may appear in the initial assignment problem. The author is going to use the same mechanisms but have a parameter change each time.

#### 1. All Officers Have the Same First Two Preferences

The author assumes that all officers have the same priority preferences, i.e., a position abroad. If the positions abroad are P2 and P3 then a potential table with the officers' annual preference list would be as follows:

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P5	P9	P1	O9:	Р3	P2	P9	P8	P1
O2:	P3	P2	P4	P8	P6	O10:	Р3	P2	P1	P4	P10
O3:	P3	P2	P8	P1	P7	O11:	P3	P2	P1	P10	P7
O4:	P3	P2	P1	P4	P10	O12:	P3	P2	P1	P8	P10
O5:	P3	P2	P8	P4	P1	O13:	Р3	P2	P9	P4	P10
O6:	Р3	P2	P8	P4	P6	O14:	P3	P2	P1	P6	P5
O7:	P3	P2	P5	P1	P4	O15:	Р3	P2	P9	P4	P7
O8:	P3	P2	P9	P8	P4						

The first two preferences of each officer change (become P2, P3 and vice versa) and the remaining three stay in the previous order. Also, each position's priorities and the table with positions' preferences remain the same. The results are as follows:

• Priority mechanism (break the ties in favor of positions)

Potential Match	O's position in Q(P)		P's position in Q(O)		Priority Number
P2, O4	1	X	1	II	1
P8, O6	1	X	3		3
P3, O5	4	X	1	Ш	4
P4, O10	1	X	4	II	4
P7, O3	1	X	5	=	5
P9, O13	3	X	3	II	9
P5, O1	3	X	3	=	9
P1, O11	3	X	3	=	9
P10, O12	6	X	5	=	30
P1, O9	6	X	5	=	30
P6, O14	11	X	4	=	44

• Deferred Acceptance (DA) mechanism

Matching based on officers' preferences:

P1	P2	Р3	P4	P5	Р6	P7	P8	Р9	P10
09, 011	04	06	010	01	014	03	012	013	02

Matching based on positions' requirements:

01	O2	O3	04	O5	06	07	08	09	010	011	O12	013	014	015
P5	-	P7	P2	P10	P3	P8	ı	P1	P4	P1	ı	P9	P6	-

• Top Trading Cycle mechanism

Matching under TTC in favor of positions:

By evaluating the results the author sees that in most cases (except priority mechanism) P2 and P3, which were the "apples of discord" for the officers, are covered by O4 and O6 respectively. It is important to mention that these two officers were the first choices of P2 and P3 as well.

# 2. Few Positions are not Preferred by any Officer

The author assumes that P5, P7 and P9 are not in the preference list of any officer. In such a case a potential table with the officers' annual preference list would be as follows:

	$1^{st}$	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	$2^{nd}$	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P2	P8	P10	P1	O9:	P2	P3	P8	P1	P6
O2:	P4	P8	P6	P10	P3	O10:	P3	P2	P1	P4	P10
O3:	P3	P2	P8	P1	P10	O11:	P2	P1	P10	P8	P6
O4:	P2	P1	P4	P10	P6	O12:	P1	P6	P2	P3	P10
O5:	P8	P4	P1	P6	P3	O13:	P3	P2	P10	P4	P6
O6:	P8	P4	P6	P1	P10	O14:	P3	P2	P1	P6	P4
O7:	P2	P10	P1	P4	P6	O15:	P10	P4	P2	P6	P8
O8:	P10	P8	P4	P6	P2						

Each position's priorities and the table with positions' preferences remain the same. The results are as follows:

• Priority mechanism (break the ties in favor of positions)

In such a case, the Priority mechanism is not useful because priorities for the pairs that contain the specific positions (zero priority number) cannot be assigned. Therefore, the generation of matches can be done but it will not include P5, P7 and P10, and another mechanism must be applied in order to cover those positions.

Deferred Acceptance (DA) mechanism
 Matching based on officers' preferences:

Following this mechanism P5, P7 and P9 are not assigned to an officer. The mechanism generates matches for the other positions.

#### Matching based on officers' preferences:

The assignment takes place regularly and it concerns all positions. The matching pairs are the following:

01	O2	O3	04	O5	06	O7	O8	09	O10	011	O12	013	O14	015
P3	-	P10	P2	P7	P8	-	P5	-	P4	P1	P6	P9	-	P1

#### • Top Trading Cycle mechanism

The assignment of P5, P7 and P9 is not possible.

By evaluating the results it can be seen that only by using Deferred Acceptance (based on positions' preferences) one could achieve the matching of the positions that do not exist in any officer's preference list.

# 3. All Officers Have Exactly the Same Preferences

The author assumes that all officers have exactly the same preferences, which means that five positions are not preferred by any officer, i.e., P1, P2, P4, P7 and P9. Such an extreme case is an extension of the previous one; a potential table with the officers' annual preference list would be as follows:

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
O1:	P3	P5	P8	P6	P10	O9:	P3	P5	P8	P6	P10
O2:	P3	P5	P8	P6	P10	O10:	P3	P5	P8	P6	P10
O3:	P3	P5	P8	P6	P10	O11:	P3	P5	P8	P6	P10
O4:	P3	P5	P8	P6	P10	O12:	P3	P5	P8	P6	P10
O5:	P3	P5	P8	P6	P10	O13:	P3	P5	P8	P6	P10
O6:	P3	P5	P8	P6	P10	O14:	P3	P5	P8	P6	P10
O7:	P3	P5	P8	P6	P10	O15:	P3	P5	P8	P6	P10
O8:	P3	P5	P8	P6	P10						

Each position's priorities and the table with positions' preferences remain the same. The results are as follows:

• Priority mechanism- Deferred Acceptance based on officers' preferences-Top Trading Cycle mechanism In such an extreme case, these mechanisms are not useful because they cannot assign priorities for the non-preferable positions P1, P2, P4, P7 and P9. Therefore, the generation of matches can be done only for P3, P5, P6, P8 and P10.

Deferred Acceptance based on positions' preferences
 This is the only mechanism that may (but not for sure) generate matches
 for all positions. The matching pairs in this specific example are the following:

01	O2	O3	O4	O5	06	O7	08	O9	O10	011	O12	O13	O14	015
-	-	P10	P3	P7	P8	P2	P4	-	P6	P1	P6	P9	-	P1

If for example the non-preferable positions were P1, P4, P5, P7 and P10, this mechanism would not generate matching for all positions. Generally, in such a case, if all positions are matched except two or more non-preferable positions, which are being deferred for the same officer, then the mechanism does not have a solution.

# 4. All Positions Have the Same First Three Preferences but in a Different Order

The author assumes that all positions have the same priority preferences, so they prefer the same three officers who possess a master's degree, have the greatest level of experience and perfect evaluation reports. Therefore, the three officers have exactly the same skills and their rank in the positions' preference list can vary. If these officers are O2, O9 and O15 a potential table with the positions' preference list would be as follows:

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	09	015	O2	O4	01	06	O3	011	O5	O7	O13	O14	O12	08	O10
P2:	O2	09	O15	06	O7	O1	O3	O4	011	O5	O14	O12	08	O10	O13
P3:	O15	O2	09	O5	011	O4	06	O7	O3	O12	01	O10	08	O13	O14
P4:	O15	O2	O9	O12	01	011	O4	O10	06	O13	O14	O7	08	O5	O3
P5:	O9	015	O2	O10	O13	06	O4	O12	011	O8	01	O7	O5	O3	O14
P6:	O9	015	O2	013	O1	O10	06	08	011	O4	O14	O12	O3	O7	O5
P7:	O15	09	O2	O8	O3	O14	O10	O13	O7	O12	06	O4	015	011	01
P8:	O9	O2	O15	01	O6	O7	O10	O13	O12	O8	O14	O5	O4	O3	O2
P9:	O2	015	O9	O8	O12	O5	O14	O7	O13	011	O4	O10	01	O3	O6
P10:	O2	09	015	O7	O3	O12	O8	O14	O13	O10	O5	011	06	01	O4

The first three preferences of each position change (become O2, O9 and O15). The rest remain the same except the previous first priorities of each position, which are put in O2, O9, and O15's previous positions. In addition, each position's priorities and the table with officers' preferences remain the same. The results are as follows:

• Priority mechanism (break the ties in favor of positions)

Potential Match	O's position in Q(P)		P's position in Q(O)		Priority Number
P4, O15	1	X	2	Ш	2
P9, O8	4	X	1	Ш	4
P10, O2	1	X	5	Ш	5
P2, O7	5	X	1	Ш	5
P8, O6	5	X	1	Ш	5
P1, O4	4	X	2	Ш	8
P3, O3	9	X	1	Ш	9
P1, O12	13	X	1	Ш	13
P5, O1	11	X	3	Ш	33
P6, O14	11	X	4	Ш	44
P7, O5	13	X	4	=	52

• Deferred Acceptance (DA) mechanism

Matching based on officers' preferences:

P1	P2	P3	P4	P5	P6	P7	Р8	Р9	P10
04, 011	09	03	02	01	08	05	06	015	012

Matching based on positions' preferences:

01	O2	O3	04	O5	06	07	08	09	010	011	O12	013	014	015
P1	P4	-	P1	P3	P8	P10	P7	P2	P5	-	P6	-	-	P9

• Top Trading Cycle mechanism

<u>Top Trading Cycle mechanism in favor of positions:</u>

By evaluating the results it can be seen that in most cases O2, O9 and O15, which were the "apples of discord" for the positions, are assigned to P4, P2 and P9 respectively. It is important to mention that there is no (1, 1) matching. That happens because even though P4, P2, and P9 are the first priorities of O2, O9 and O15 respectively, each officer is the second priority of the assigned position.

# 5. All Positions Have Exactly the Same First Five Preferences

The author assumes that five officers are qualified in such a way that they are the first choices of each position; that means that each position has exactly the same (first five) preferences, i.e., O1, O2, O4, O7 and O9 with this specific order. Such an extreme case is an extension of the previous one, and number five is chosen because each officer can fill five preferable positions in his/her annual list. A potential table with the positions' preference list would be as follows:

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	01	O2	O4	O7	09	06	O3	011	O5	O7	013	014	O12	08	O10
P2:	01	O2	O4	O7	09	015	O3	06	011	O5	O14	012	08	O10	013
P3:	01	O2	O4	O7	09	O5	06	O15	O3	O12	011	010	08	013	O14
P4:	01	O2	O4	O7	09	011	O12	O10	06	013	O14	015	08	O5	O3
P5:	01	O2	O4	O7	09	06	O15	O12	011	O8	013	O10	O5	O3	O14
P6:	O1	O2	O4	O7	09	O10	06	08	011	O14	O15	012	O3	013	O5
P7:	01	O2	O4	O7	09	O14	O10	O13	O15	O12	06	03	O5	011	08
P8:	01	O2	O4	O7	09	015	O10	O13	O12	08	O14	06	O5	O3	011
P9:	01	O2	O4	O7	09	O5	O14	O12	O13	011	O15	O10	O8	O3	06
P10:	01	O2	O4	O7	09	O12	08	O14	011	O10	O5	013	06	O15	04

Each position's priorities and the table with officers' preferences remain the same. The results are as follows:

• Priority mechanism (break the ties in favor of positions)

Potential Match	O's position in Q(P)		P's position in Q(O)		Priority Number
P3, O1	1	X	1	Ш	1
P4, O2	2	X	3	Ш	2
P1, O4	3	X	2	Ш	6
P5, O7	2	X	4	Ш	8
P9, O9	5	X	2		10
P8, O6	12	X	1		12
P1, O12	13	X	1	Ш	13
P2, O3	7	X	2	Ш	14
P7, O15	9	X	3	Ш	27
P6, O8	8	X	4	Ш	32
P10, O10	10	X	5	Ш	50

• Deferred Acceptance (DA) mechanism

Matching based on officers' preferences:

P1	P2	Р3	P4	P5	P6	P7	Р8	Р9	P10
03, 011	04	01	02	07	06	015	012	09	011

# Matching based on positions' preferences:

01	02	03	04	05	06	07	08	09	010	011	012	013	014	015
P3	P4	P1	P2	-	P1	P5	-	P9	P6	-	P10	-	P7	P8

• Top Trading Cycle mechanism

<u>Top Trading Cycle mechanism in favor of positions:</u>

In this case it is impressive to mention that the five preferable officers (O1, O2, O4, O7 and O9) are assigned to the same positions using any mechanism (except O4 in the Priority mechanism). Furthermore, in any (extreme) case where the positions' preference list has more than the above mentioned five officers in the exact order, the assignment of the first five officers remain the same.

6. All Officers Have Exactly the Same Preferences and All Positions Have Exactly the Same First Five Preferences

A potential officer's list and a potential	positions' p	preference li	ist are as follows:
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	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>		1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>
01:	P3	P5	P8	P6	P10	O9:	P3	P5	P8	P6	P10
O2:	P3	P5	P8	P6	P10	O10:	P3	P5	P8	P6	P10
O3:	P3	P5	P8	P6	P10	O11:	P3	P5	P8	P6	P10
O4:	P3	P5	P8	P6	P10	O12:	P3	P5	P8	P6	P10
O5:	P3	P5	P8	P6	P10	O13:	P3	P5	P8	P6	P10
O6:	P3	P5	P8	P6	P10	O14:	P3	P5	P8	P6	P10
O7:	P3	P5	P8	P6	P10	O15:	P3	P5	P8	P6	P10
O8:	P3	P5	P8	P6	P10						

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	7 <sup>th</sup>	8 <sup>th</sup>	9 <sup>th</sup>	10 <sup>th</sup>	11 <sup>th</sup>	12 <sup>th</sup>	13 <sup>th</sup>	14 <sup>th</sup>	15 <sup>th</sup>
P1:	01	O2	O4	O7	O9	06	O3	011	O5	O7	O13	O14	O12	08	O10
P2:	01	O2	O4	O7	O9	015	O3	06	011	O5	O14	O12	08	O10	O13
P3:	01	O2	O4	O7	O9	O5	06	015	O3	O12	011	O10	08	013	O14
P4:	01	O2	O4	O7	O9	011	O12	O10	06	013	O14	O15	08	O5	O3
P5:	01	O2	O4	O7	O9	06	O15	O12	011	O8	O13	O10	O5	O3	O14
P6:	01	O2	O4	O7	O9	O10	O6	08	011	O14	O15	O12	O3	O13	O5
P7:	01	O2	O4	O7	O9	O14	O10	O13	O15	O12	06	O3	O5	011	O8
P8:	01	O2	O4	O7	O9	015	O10	O13	O12	O8	O14	O6	O5	O3	011
P9:	01	O2	04	O7	09	O5	O14	O12	O13	011	O15	O10	08	O3	06
P10:	01	O2	O4	O7	O9	O12	O8	O14	011	O10	O5	O13	06	O15	O4

In such a case the only mechanism that may be applied is the Deferred Acceptance mechanism based on positions' preferences, but there is no guarantee that a final matching will take place. In the positions' table, if in the sixth preference two

positions prefer the same officer that means that there is no solution because there is no way of deference. In this particular example that does not happen, thus the final matching is as follows:

01	O2	O3	04	O5	06	O7	08	09	O10	011	O12	013	O14	015
P3	P5	P1	P8	P9	P1	P6	-	P10	P6	P4	-	-	P7	P2

However, if for example any position (except P3, P5, P6, P8, and P10 which are matched at the first five steps) had as its sixth preference the same one with P1, which is O6, then the mechanism would not be continued; similarly if any position had as its sixth preference the same one with P2, which is O15, etc. Therefore, in this case the matching mechanism ends in the sixth position preference, successfully or not.

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#### V. EVALUATION OF RESULTS - SCORING METHODS

#### A. APPLICATION OF SCORING METHODS

In the previous chapters the author thoroughly described two-sided matching theory and focused on three of its mechanisms—Priority, Deferred Acceptance and Top Trading Cycle. The purpose was to apply them in the Hellenic Navy's assignment process and extract useful conclusions. For that reason, the author assumed an example where 15 Hellenic Navy officers (O1, O2...,O15) were eligible as candidates for 10 positions (P1, P2...,P10). The author tried to reflect a real situation as possible by:

- Creating a table that contained officers' annual preference list based on the positions' division in groups of similar nature; that means that every officer who had as first priority a specific position had as second and third priorities the positions of the same group.
- Creating a table that contained each position's preference list based on the eligible officers; officers were divided in groups according to specific characteristics, namely previous performance, possession of a master's degree (as an indicator of their educational level) and experience.
- Trying to apply many different cases; for that reason the author assumed that the positions had various priorities.
- Assuming that the above mentioned criteria were not mutually exclusive, so an officer could be qualified for more than one simultaneously.
- Considering that the majority of the positions needed one officer so as to be covered but there was a position that needed two officers.

The final matching between officers and positions for each mechanism were:

#### • Priority mechanism

Potential Match	O's position in Q(P)		P's position in Q(O)		Priority Number
P2, O4	1	X	1		1
P8, O6	1	X	1	II	1
P3, O1	3	X	1		3
P4, O10	1	X	4		4
P9, O8	4	X	1		4
P5, O12	1	X	5		5
P7, O3	1	X	5	=	5
P1, O11	3	X	2		6
P10, O5	2	X	5		10
P1, O15	2	X	5		10
P6, O9	6	X	5		30

# Deferred Acceptance (DA) mechanism Matching based on officers' preferences:

P1	P2	Р3	P4	P5	P6	P7	P8	Р9	P10
015	04	01	010	012	08	03	06	013	05

# Matching based on positions' requirements:

01	O2	O3	04	O5	06	O7	O8	09	O10	011	O12	013	014	015
P3	-	P7	P2	P10	P8	-	P6	-	P4	P1	P5	P9	-	P1

# • Top Trading Cycle (TTC) mechanism

# Matching under TTC in favor of positions:

In the previous chapter the author evaluated and compared the results according to two-sided matching theory. He mentioned the common points and explained the differences that resulted; that is, he focused on theory. That is one point of view. The other point of view says that in order to choose the best applicable mechanism a scoring method is needed, a method based on mathematical formulas or statistical parameters that will indicate the appropriate mechanism so as to apply it in the Hellenic Navy's assignment process. The author will try to achieve this by using the following three main scoring methods (nine versions in total).

#### 1. Arithmetic Mean

The first scoring method is based on the arithmetic mean and the author uses this statistical parameter in four different ways. The first version is based on the sum of each officer's place in the position's preference list, for every matching pair; the second one is

based on the sum of each position's place in the officer's preference list for every matching pair; the third one is based on both the above mentioned sums while the fourth one is based on the sum of the partial scores (partial score is the product between the number that represents the officer's place in the position's preference list and the number that represents the position's place in the officer's preference list for every matching pair). Comparing the final scores of all mechanisms (Deferred Acceptance is consider as one mechanism because its two versions have identical final matching) the author concludes that the mechanism with the lowest final score is the "winner." Thus, the results by applying this method in four different versions are as follows:

Using the sum of officer's place in the position's preference list:
 Priority mechanism

$$(1+1+3+1+4+1+1+3+2+2+6)/11=25/11=2.27$$

Deferred Acceptance (DA) mechanism

$$(3+2+1+3+1+1+3+1+1+3+2)/11=21/11=1.9$$

<u>Top Trading Cycle mechanism (TTC)</u>

$$(15+3+1+3+2+1+6+1+1+12+2)/11=47/11=4.27$$

Matching under TTC in favor of positions:

$$(3+2+1+3+2+1+6+1+1+1+2)/11=23/11=2.09$$

Using the sum of position's place in the officer's preference list:
 Priority mechanism

$$(1+1+1+4+1+5+5+2+5+5+5)/11=35/11=3.18$$

Deferred Acceptance (DA) mechanism

$$(2+5+1+1+4+5+4+5+1+3+5)/11=36/11=\overline{3.27}$$

<u>Top Trading Cycle mechanism (TTC)</u>

$$(3+2+1+1+3+5+5+5+1+1+5)/11=32/11=2.91$$

Matching under TTC in favor of positions:

$$(2+5+1+1+3+5+5+5+1+5+5)/11=38/11=3.45$$

• Using both above mentioned sums:

Priority mechanism

$$[(1+1)+(1+1)+(3+1)+(1+4)+(4+1)+(1+5)+(3+2)+(1+5)+(2+5)+$$

$$(2+5)+(6+5)]/11=\overline{[5.45]}$$

#### Deferred Acceptance (DA) mechanism

$$[(2+5)+(3+2)+(1+1)+(3+1)+(1+4)+(1+5)+(3+4)+(1+5)+(1+1)+(3+3)+(2+5)]=57/11=\boxed{5.18}$$

#### <u>Top Trading Cycle mechanism (TTC)</u>

$$[(15+3)+(3+2)+(1+1)+(3+1)+(2+3)+(1+5)+(6+5)+(1+5)+(1+1)+(12+1)+(2+5)]=79/11=\boxed{7.18}$$

Matching under TTC in favor of positions:

$$[(2+5)+(3+2)+(1+1)+(3+1)+(2+3)+(1+5)+(6+5)+(1+5)+(1+1)+(1+5)+(2+5)] = 61/11 = \boxed{5.55}$$

• Using partial scores

Priority mechanism

$$[(1x1)+(1x1)+(3x1)+(1x4)+(4x1)+(1x5)+(3x2)+(1x5)+(2x5)+(2x5)+(6x5)]/11=(1+1+3+4+4+5+5+6+10+10+30)/11=79/11=\boxed{7.18}$$

#### <u>Deferred Acceptance (DA) mechanism</u>

$$[(2x5)+(3x2)+(1x1)+(3x1)+(1x4)+(1x5)+(3x4)+(1x5)+(1x1)+(3x3)+(2x5)]/11=(10+6+1+3+4+5+12+5+1+9+10)/11=66/11=\boxed{6}$$

Top Trading Cycle mechanism (TTC)

$$[(15x3)+(3x2)+(1x1)+(3x1)+(2x3)+(1x5)+(6x5)+(1x5)+(1x1)+(12x1)+(2x5)]/11=(45+6+1+3+6+5+30+5+1+12+10)/11=134/11=\boxed{12.18}$$

Matching under TTC in favor of positions:

$$[(2x5)+(3x2)+ (1x1)+ (3x1)+(2x3)+ (1x5)+(6x5)+ (1x5)+ (1x1)+(1x5)+ (2x5)] = (10+6+1+3+6+5+30+5+1+5+10)/11 = 82/11= \boxed{7.45}$$

Comparing the extracted arithmetic means, we mention that the Deferred Acceptance (DA) mechanism is the most preferable one in the three of four versions, the priority mechanism is the second preferable while the Top Trading Cycle in favor of positions' requirements and the Top Trading Cycle mechanism come next.

#### 2. Median

There is no doubt that the arithmetic mean is the single most popular and useful measure of central location. Its greatest disadvantage is that it is very sensitive to "extreme values." Therefore, we are going to try another method that is less sensitive to extreme values to measure the central location, the median. The median equals the observation that falls in the middle if all the observations are placed in order. In a case of an even number of observations the median equals the average of the two observations that fall in the middle.<sup>27</sup> The results by applying this method using the partial scores are the following:

#### • Priority mechanism:

$$1^{st}$$
 obs:  $1x1=1$ 
 $2^{nd}$  obs:  $1x1=1$ 
 $3rd$  obs:  $3x1=3$ 
 $4^{th}$  obs:  $1x4=4$ 
 $5^{th}$  obs:  $4x1=4$ 
 $6^{th}$  obs:  $1x5=5$ 
 $7^{th}$  obs:  $3x2=6$ 
 $8^{th}$  obs:  $1x5=5$ 
 $9^{th}$  obs:  $2x5=10$ 
 $10^{th}$  obs:  $2x5=10$ 
 $11^{th}$  obs:  $6x5=30$ 

 The order is:  $1, 1, 3, 4, 4, 5, 5, 6, 10, 10, 30$ 
 $\longrightarrow$  median is  $5$ 

## • Deferred Acceptance (DA) mechanism:

$$1^{st}$$
 obs:  $2x5=10$   $2^{nd}$  obs:  $3x2=6$   $3^{rd}$  obs:  $1x1=1$   $4^{th}$  obs:  $3x1=3$   $5^{th}$  obs:  $1x4=4$   $6^{th}$  obs:  $1x5=5$   $7^{th}$  obs:  $3x4=12$   $8^{th}$  obs:  $1x5=5$   $9^{th}$  obs:  $1x1=1$   $10^{th}$  obs:  $3x3=10$   $11^{th}$  obs:  $2x5=10$ 

The order is: 1, 1, 3, 4, 5, 5, 6, 10, 10, 10, 12  $\longrightarrow$  median is  $\boxed{5}$ 

#### • <u>Top Trading Cycle mechanism (TTC):</u>

$$1^{st}$$
 obs:  $15x3=45$   $2^{nd}$  obs:  $3x2=6$   $3^{rd}$  obs:  $1x1=1$   $4^{th}$  obs:  $3x1=3$   $5^{th}$  obs:  $2x3=6$   $6^{th}$  obs:  $1x5=5$   $7^{th}$  obs:  $6x5=30$   $8^{th}$  obs:  $1x5=5$   $9^{th}$  obs:  $1x1=1$   $10^{th}$  obs:  $12x1=10$   $11^{th}$  obs:  $2x5=10$ 

The order is: 1, 1, 3, 5, 5, 6, 6, 10, 10, 30, 45  $\longrightarrow$  median is 6

Matching under TTC in favor of positions:

$$1^{st}$$
 obs:  $2x5=45$ 
 $2^{nd}$  obs:  $3x2=6$ 
 $3^{rd}$  obs:  $1x1=1$ 
 $4^{th}$  obs:  $3x1=3$ 
 $5^{th}$  obs:  $2x3=6$ 
 $6^{th}$  obs:  $1x5=5$ 
 $7^{th}$  obs:  $6x5=30$ 
 $8^{th}$  obs:  $1x5=5$ 
 $9^{th}$  obs:  $1x1=1$ 
 $10^{th}$  obs:  $1x5=5$ 
 $11^{th}$  obs:  $2x5=10$ 

 The order is:  $1, 1, 3, 5, 5, 5, 6, 6, 10, 30, 45$ 
 $\longrightarrow$  median is  $\boxed{5}$ 

The results show that three of the four mechanisms have exactly the same median; that means that this scoring method is not appropriate in order to evaluate the two-sided matching mechanisms.

#### 3. Standard Deviation

The third scoring method is based on statistics too and it concerns the standard deviation. This measure shows how far a set of numbers is spread out. The standard deviation equals to the square root of a fraction that has as a numerator the sum of all squared differences between each value and the mean, and as a denominator the number of "observations."<sup>27</sup> The author applies again four versions of this scoring method by using the same parameters as he did in the first one (arithmetic mean). The mechanism with the lowest standard deviation will be the preferable one. Thus, the results by applying this method are the following:

• Using the sum of officer's place in the position's preference list:

#### Priority mechanism

$$Mean = 2.27$$

$$[(1-2.27)^2 + (1-2.27)^2 + (3-2.27)^2 + (1-2.27)^2 + (4-2.27)^2 + (1-2.27)^2 + (1-2.27)^2 + (1-2.27)^2 + (2-2.27)^2 + (2-2.27)^2 + (6-2.27)^2]/11 =$$

$$(1.61 + 1.61 + 0.53 + 1.61 + 2.99 + 1.61 + 1.61 + 0.53 + 0.07 + 0.07 + 13.91)/11 = 26.15/11 = 2.38$$

$$\sigma = \sqrt{2.38} = 1.54$$

# Deferred Acceptance (DA) mechanism

$$Mean = 1.9$$

$$[(3-1.9)^2 + (2-1.9)^2 + (1-1.9)^2 + (3-1.9)^2 + (1-$$

$$(1.21 + 0.01 + 0.81 + 1.21 + 0.81 + 0.81 + 1.21 + 0.81 + 0.81 + 1.21 + 0.01)/11 = 8.91/11 = 0.81$$

# $\sigma = \sqrt{0.81} = 0.9$

#### <u>Top Trading Cycle mechanism (TTC)</u>

$$Mean = 4.27$$

$$[(15-4.27)^2+(3-4.27)^2+(1-4.27)^2+(3-4.27)^2+ (2-4.27)^2+(1-4.27)^2+(6-4.27)^2+(1-4.2$$

$$(115.13 + 1.61 + 10.69 + 1.61 + 5.15 + 10.69 + 2.99 + 10.69 + 10.69 + 59.75 + 5.15)/11 = 234.15/11 = 21.29$$

$$\sigma = \sqrt{21.29} = 4.61$$

Matching under TTC in favor of positions:

Mean = 
$$2.09$$

$$[(3-2.09)^2 + (2-2.09)^2 + (1-2.09)^2 + (3-2.09)^2 + (2-2.09)^2 + (1-2.09)^2 + (6-2.09)^2 + (1$$

$$(0.83 + 0.01 + 1.19 + 0.83 + 0.01 + 1.19 + 15.29 + 1.19 + 1.19 + 1.19 + 0.01) / 11 = 22.93 / 11 = 2.08$$

$$\sigma = \sqrt{2.08} = \boxed{1.44}$$

• Using the sum of position's place in the officer's preference list:

#### Priority mechanism

Mean = 3.18

$$[(1-3.18)^2 + (1-3.18)^2 + (1-3.18)^2 + (4-3.18)^2 + (1-3.18)^2 + (5$$

$$(4.75 + 4.75 + 4.75 + 0.67 + 4.75 + 3.31 + 3.31 + 3.31 + 3.31 + 3.31)/11 = 37.61/11 = 3.42$$

$$\sigma = \sqrt{3.42} = \boxed{1.85}$$

#### <u>Deferred Acceptance (DA) mechanism</u>

Mean = 3.27

$$[(2-3.27)^2 + (5-3.27)^2 + (1-3.27)^2 + (1-3.27)^2 + (4-3.27)^2 + (5-3.27)^2 + (4-3.27)^2 + (5-3.27)^2 + (4-3.27)^2 + (5-3.27)^2 + (1-3.27)^2 + (5-3.27)^2 + (1$$

$$(1.61+2.99+5.15+5.15+0.53+2.99+0.53+2.99+5.15+0.07+2.99)/11=30.15/11=2.74$$

$$\sigma = \sqrt{2.74} = \boxed{1.66}$$

#### <u>Top Trading Cycle mechanism (TTC)</u>

Mean = 2.91

$$[(3-2.91)^2 + (2-2.91)^2 + (1-2.91)^2 + (1-2.91)^2 + (3-2.91)^2 + (5-2.91)^2 + (5-2.91)^2 + (5-2.91)^2 + (1$$

$$(0.008+0.83+3.65+3.65+0.008+4.37+4.37+4.37+3.65+3.65+4.37)/11=32.93/11=2.99$$
  
$$\sigma = \sqrt{2.99} = \boxed{1.73}$$

Matching under TTC in favor of positions:

$$Mean = 3.45$$

$$[(2-3.45)^2 + (5-3.45)^2 + (1-3.45)^2 + (1-3.45)^2 + (3-3.45)^2 + (5$$

$$(2.1+2.4+6+6+0.2+2.4+2.4+2.4+6+2.4+2.4)/11=34.7/11=3.15$$

$$\sigma = \sqrt{3.15} = \boxed{1.77}$$

• Using both above mentioned sums:

#### Priority mechanism

$$Mean = 5.45$$

$$[(2-5.45)^2 + (2-5.45)^2 + (4-5.45)^2 + (5-5.45)^2 + (5-5.45)^2 + (6-5.45)^2 + (6-5.45)^2 + (6-5.45)^2 + (7-5.45)^2 + (7-5.45)^2 + (11-5.45)^2]/11 =$$

$$(11.9+11.9+2.1+0.2+0.2+0.3+0.2+0.3+2.4+2.4+30.8)/11 = 62.7/11 = 5.7$$

$$\sigma = \sqrt{5.7} = 2.39$$

#### Deferred Acceptance (DA) mechanism

Matching based on positions' requirements:

$$Mean = 5.18$$

$$[(7-5.18)^2 + (5-5.18)^2 + (2-5.18)^2 + (4-5.18)^2 + (5-5.18)^2 + (6-5.18)^2 + (7-5.18)^2 + (6-5.18)^2 + (6-5.18)^2 + (6-5.18)^2 + (6-5.18)^2 + (7$$

$$(3.31+0.03+10.11+1.39+0.03+0.67+3.31+0.67+10.11+0.67+3.31)/11=33.61/11=3.06$$

$$\sigma = \sqrt{3.06} = \boxed{1.75}$$

#### Top Trading Cycle mechanism (TTC)

$$Mean = 7.18$$

$$[(18-7.18)^2 + (5-7.18)^2 + (2-7.18)^2 + (4-7.18)^2 + (5-7.18)^2 + (6-7.18)^2 + (11-7.18)^2 + (6-7.18)^2 + (2-7.18)^2 + (13-7.18)^2 + (7-7.18)^2]/11 =$$

$$(117.07 + 4.75 + 26.83 + 10.11 + 4.75 + 1.39 + 14.59 + 1.39 + 26.83 + 33.87 + 0.03)/$$
 11 =  $241.61/$  11 =  $21.96$ 

$$\sigma = \sqrt{21.96} = 4.69$$

Matching under TTC in favor of positions:

$$Mean = 5.55$$

$$[(7-5.55)^{2} + (5-5.55)^{2} + (2-5.55)^{2} + (4-5.55)^{2} + (5-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (6-5.55)^{2} + (7-5.55)^{2}]/11 =$$

$$(2.1+0.3+12.6+2.4+0.3+0.2+29.7+0.2+12.6+0.2+2.1)/11 = 62.7/11 = 5.7$$

$$\sigma = \sqrt{5.7} = \boxed{2.38}$$

#### • Using partial scores

# Priority mechanism

Mean = 
$$7.18$$

$$[(1-7.18)^2 + (1-7.18)^2 + (3-7.18)^2 + (4-7.18)^2 + (4-7.18)^2 + (5-7.18)^2 + (6-7.18)^2 + (5-7.18)^2 + (10-7.18)^2 + (10-7.18)^2 + (30-7.18)^2]/11 =$$

$$(38.19+38.19+17.47+13.82+13.82+4.75+1.39+4.75+7.95+7.95+520.75)/11 = 669.05/11 =$$

$$60.82$$

$$\sigma = \sqrt{60.82} = 7.8$$

#### Deferred Acceptance (DA) mechanism

$$Mean = 6$$

$$[(10-6)^2 + (6-6)^2 + (1-6)^2 + (3-6)^2 + (4-6)^2 + (5-6)^2 + (12-6)^2 + (5-6)^2 + (1-6)^2 + ($$

## Top Trading Cycle mechanism (TTC)

Mean = 
$$12.18$$

$$[(45-12.18)^2 + (6-12.18)^2 + (1-12.18)^2 + (3-12.18)^2 + (6-12.18)^2 + (5-12.18)^2 + (30-12.18)^2 + (5-12.18)^2 + (1-12.18)^2 + (12-12.18)^2 + (10-12.18)^2]/11 = (1077.15+38.19+124.99+84.27+38.19+51.55+317.55+51.55+124.99+0.03+4.75)/11 = 1913.22/11 = 173.93$$

$$\sigma = \sqrt{173.93} = 13.19$$

Matching under TTC in favor of positions:

Mean = 
$$7.45$$

$$[(10-7.45)^{2} + (6-7.45)^{2} + (1-7.45)^{2} + (3-7.45)^{2} + (6-7.45)^{2} + (5-7.45)^{2} + (5-7.45)^{2} + (5-7.45)^{2} + (5-7.45)^{2} + (10-7.45)^{2} + (10-7.45)^{2}]/11 =$$

$$(6.5+2.1+41.6+19.8+2.1+6+508.5+6+41.6+6+6.5)/11 = 646.722/11 = 58.79$$

$$\sigma = \sqrt{58.79} = \boxed{7.67}$$

Comparing the extracted standard deviations, the author notices that again the Deferred Acceptance (DA) mechanism with matching based on positions' requirements is clearly the most preferable one. The Priority mechanism and the Top Trading Cycle in favor of positions' requirements have almost the same standard deviation (but this time TTC is slightly better), while the Top Trading Cycle mechanism seems to be far away.

The results that derived from the application of the scoring methods lead to important conclusions as far as the effectiveness of each mechanism is concerned:

- The application of multiple scoring methods offers the potentiality of choosing the most preferable according to specific criteria that someone will set; for example choosing a mechanism with the smallest Standard deviation will minimize variation or spread among the matches, the partial scores could be preferred from a sum method in order to penalize a big mismatch etc.
- Median scoring method was proved totally ineffective even though in comparison with the arithmetic mean method has the advantage of non sensitivity in extreme values. That happens because three out of four twosided mechanisms produced the same final score, thus more criteria for the final choice are needed.
- Standard deviation is the most complex but not necessarily the most accurate method.
- There's a common result according to which the Deferred Acceptance is the most preferable algorithm in 8 out of 9 versions of scoring methods.
- Since the choice of the most preferable method proved easy, the choice of the alternative one had difficulties. The reason is that Top Trading Cycle

in favor of positions and the Priority mechanism seem to present similar results and their effectiveness depend on the applied scoring method; thus Priority mechanism seems to be more effective in arithmetic mean scoring method while Top Trading Cycle in favor of positions seems to be more effective in Standard deviation method. By the time that Standard deviation method is a more complex, Top Trading Cycle in favor of positions could be considered as the second choice for quantitative evaluation.

• Top Trading Cycle (according to Abdulkadiroglu and Sonmez<sup>17</sup>) produced the highest scores (not effective) in 7 out of 9 methods. The two scoring methods in which it gave good results were those that focused on the officer's preference list; that seems logical if we take into account that the specific mechanism's matching is in students/officers favor.

#### VI. SUMMARY - CONCLUSIONS - RECOMMENDATIONS

#### A. SUMMARY AND CONCLUSIONS

Assignment is the process according to which the organization's personnel is employed in such a way that covers both the organization's needs and the personnel's preferences. The design and the application of an appropriate assignment process within an organization is not a simple procedure and it depends on various parameters. This thesis demonstrates the complexity of such a task focusing on crucial parameters like the human factor (appeared in many ways such as human decision making/skills/ educational level/ past performance/experience/preferences), positions' specific requirements, organizational structure, etc.

In addition, this thesis focused on the assignment process in a military organization and specifically the Hellenic Navy, indicating the unique qualitative and quantitative features, differences and gaps (like the amount of personnel that has to be assigned, the frequency of the assignment process' application, the role of hierarchy, etc.) that make it a large-scale, complex process that is difficult to "solve."

In response to the weaknesses regarding the assignment process in the Hellenic Navy, this thesis developed two-sided matching theory as an alternative approach. Two-sided matching refers to the presence of two distinct groups of agents and the bilateral nature of exchange where every agent of each group seeks to match with his/her most preferable agent from the other group. Two main categories of models exist, the one-to-one model (each agent is matched with just one agent, i.e., marriage) and the many-to-one model (the agents of one group seek to be matched with many agents of the other group, i.e., college admission).

The application of two-sided matching theory has many advantages.

- It tries to balance the preferences of both involved parties.
- The outcome is stable and can be an optimal one too.
- In case of ties alternative solutions are resulted.

• The consideration of both sides' preferences improves the welfare of the involved parties.

Disadvantages also appear (like the necessity of having complete preference lists from both sides and the uncertainness of matching each agent with his/her most preferable pair), but their weight cannot eliminate the theory's importance.

Three two-sided matching mechanisms—Priority, Deferred Acceptance (based on officers' preferences and positions' preferences), and Top Trading Cycle (according to <sup>17</sup> and in favor of positions)—used for assignment purposes in many organizations were chosen. Their functions are described and analyzed focusing on the different attributes that each mechanism has. The specific attributes of each mechanism are shown in the following table:

Attributes/ Mechanisms	Priority	Gale- Shapley Deferred Acceptance	Top Trading Cycle
Stability	No	Yes	No
Strategy proof	No	Yes	Yes
Pareto Efficient	No	No*	Yes
Complete elimination of	No	Yes*	No
justified envy			
*: in conflict			

The main goal of this thesis was to apply two-sided matching theory and specifically the above mentioned mechanisms in the Hellenic Navy's assignment process in order to achieve an effective matching between officers and positions. For that reason the author assumed an example where fifteen Hellenic Navy officers (O1, O2 ..., and O15) were eligible as candidates for ten positions (P1, P2... P10). He tried to reflect a real situation by taking into consideration the totally unique nature of the military environment that in several cases overthrows the theory, as well as the specific characteristics of the Hellenic Navy.

Evaluating and comparing the results from a qualitative point of view, the author concludes that:

• The three mechanisms demonstrated six common pairs (officer-position) out of eleven.

- Priority mechanism's final matching is unstable; that is, there were pairs of agents that had not been assigned to each other even though they preferred it rather than their match. The instability means that positions that require specific criteria were covered by officers that were not qualified enough.
- The two versions of the Deferred Acceptance mechanism produced identical and stable matching.
- The final matching of Top Trading Cycle's both versions were unstable.
- In general, from the positions' perspective the final pairs extracted from the Deferred Acceptance mechanism were likely to be more favorable, while from the officers' perspective the results extracted from the three mechanisms were similar and the differences insignificant.

However, in order to choose the best applicable mechanism a quantitative evaluation was also needed; therefore, the author applied three main scoring methods (with variants) based on mathematical formulas or statistical parameters whose results indicate the appropriate mechanism so as to apply it in the Hellenic Navy's assignment process. The chosen main scoring methods were:

- Arithmetic mean.
- Median.
- Standard deviation.

Arithmetic mean and Standard deviation scoring methods are applied in four different versions each. The first version was based on the sum of each officer's place in the position's preference list, for every matching pair; the second one was based on the sum of each position's place in the officer's preference list for every matching pair; the third one was based on both the above mentioned sums while the fourth one was based on the sum of the partial scores (partial score is the product between the number that represents the officer's place in the position's preference list and the number that represents the position's place in the officer's preference list for every matching pair).

The application of the scoring methods gave the author the opportunity to extract significant conclusions as follows:

- The Deferred Acceptance mechanism is proved as the most preferable algorithm in 8 out of 9 versions of scoring methods.
- Top Trading Cycle in favor of positions and the Priority mechanism seem

to present similar results; Priority mechanism seems to be more effective in arithmetic mean scoring method while Top Trading Cycle in favor of positions seems to be more effective in Standard deviation method.

- Top Trading Cycle (according to Abdulkadiroglu and Sonmez<sup>17</sup>) produced the highest scores (not effective) in 7 out of 9 methods. The two scoring methods in which it gave good results were those which took into account the position's place in the officer's preference list.
- Standard deviation is the most complex but not necessarily the most accurate method.
- Median scoring method was proved totally ineffective because three out of four two-sided mechanisms produced the same final score.

The extracted conclusions from the application of the above mentioned scoring methods can be abstracted in the following table (where the numbers 1–4 reflect the rank that each mechanism achieved according to its final score; that is, 1 is the mechanism with the lowest score (so the "winner"), 2 with the second lowest score, etc.):

Table 22. Quantitative Evaluation of the Two-Sided Mechanisms (Scoring Methods)

Two-sided mechanism/ Scoring method		Priority	Gale and Shapley's Deferred Acceptance	Top Trading Cycle	
				According to Abdulkadiroglu and Sonmez	In favor of positions
rithmetic Mea	Sum of officer's place in the position's preference list	3	1	4	2
	Sum of position's place in the officer's preference list	2	3	1	4
	Both sums	2	1	4	3
	Partial scores	2	1	4	3
Median		1	1	4	1
andard Deviati	Sum of officer's place in the position's preference list	3	1	4	2
	Sum of position's place in the officer's preference list	4	1	2	3
	Both sums	3	1	4	2
	Partial scores	3	1	4	2

The application of multiple scoring methods in order to evaluate the two-sided mechanisms in a quantitative way intends to present the potentiality of choosing the most appropriate method according to specific criteria. For example, choosing a mechanism with the smallest standard deviation will minimize variation or spread among the matches, the partial scores could be preferred from a sum method in order to penalize a big mismatch etc.

The following figure offers a visual image that combines the qualitative and quantitative evaluation of the mechanisms' results as they described above.

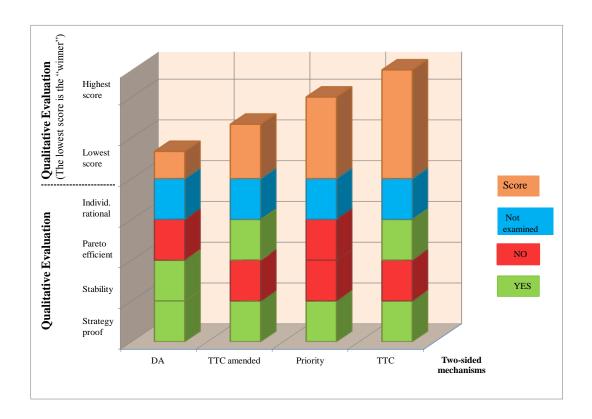


Figure 21. Qualitative and quantitative evaluation

Furthermore, a few more examples from the Hellenic Navy were developed covering different cases, even extreme ones, so as to realize the function and analyze the results of the specific two-sided mechanisms under various circumstances.

#### B. RECOMMENDATIONS

The fact is there is a need for a more efficient and effective assignment process in the Hellenic Navy. An assignment process based on specific algorithms, which are less time-consuming, that takes into account the attributes of officers including their preferences and the positions' requirements would have significant positive effects. These positive effects would be for the human part of the organization (like increased performance, morale, satisfaction and maybe retention) and the organization in general (like covering the positions with qualified personnel, better performance, etc.).

This thesis provided two-sided matching theory as an alternative process for assigning the officers of the Hellenic Navy (and personnel in general) to positions. The advantages of the two-sided matching theory's particular mechanisms and the extracted conclusions from the above described examples showed that Hellenic Navy's Department of Personnel may want to seriously consider the adoption of a two-sided matching mechanism as the primary method of assignment. However, the implementation of such a process cannot be the same for any organization; specific characteristics and functions of the Hellenic Navy have to be taken into consideration and various modifications must take place before the application.

- First of all, the creation of a manpower database; this is necessary because it would give detailers the opportunity for direct/fast access and managing of a huge amount of information which so far is not being processed in an optimal manner or even taken into consideration at all. Also, the existence of a manpower database would make the assignment process less time-consuming, while the effort needed would be significantly eliminated.
- More attention must be paid to the design of the evaluation report form, so as to include elements of the evaluator's personality and perceptions. That would be very helpful in order to eliminate the human bias to a great degree and the form to reflect the performance of the person who is evaluated (a major parameter taken into account during the assignment process) as realistic as possible.
- It would be useful for the process if the preference list, which an officer has to fill out and submit annually, gave the officer the opportunity to add more preferable positions. That would not reduce the officer's possibility of being matched with a position of high preference but it would provide further matching options in a case where the matching with positions of

higher priority was infeasible. Furthermore, an officer's preference list with many options would be a handy tool so as to check the rationality of the final matching.

- The elimination of the times that the assignment process takes place during a year must be considered because the current situation where the assignment process is applied several times leads to an increased complexity of the process with negative results.
- Finally, it is well known that many times the application of theory in real life does not produce the expected results. In such a "sensitive" case like the assignment process in a military environment, the decision makers must be very careful because potential mistakes could prove to be harmful. Hence, it is recommended that before the implementation of two-sided matching theory in the Hellenic Navy's assignment process, a partial application in the framework of experimentation would be utilitarian and provide many useful conclusions.

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